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Combining the Magnetic Equivalent Circuit and Maxwell–Fourier Method for Eddy-Current Loss Calculation

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Abstract: In this paper, a hybrid model in Cartesian coordinates combining a two-dimensional (2-D) generic magnetic equivalent circuit (MEC) with a 2-D analytical model based on the Maxwell–Fourier method (i.e., the formal resolution of Maxwell’s equations by using the separation of variables method and the Fourier’s series) is developed. This model coupling has been applied to a U-cored static electromagnetic device. The main objective is to compute the magnetic field behavior in massive conductive parts (e.g., aluminum, magnets, copper, iron) considering the skin effect (i.e., with the eddy-current reaction field) and to predict the eddy-current losses. The magnetic field distribution for various models is validated with 2-D and three-dimensional (3-D) finite-element analysis (FEA). The study is also focused on the discretization influence of 2-D generic MEC on the eddy-current loss calculation in conductive regions. Experimental tests and 3-D FEA have been compared with the proposed approach on massive conductive parts in aluminum. For an operating point, the computation time is divided by ~ 4.6 with respect to 3-D FEA.

Keywords: eddy-current losses; experiment; hybrid model; magnetic equivalent circuit; numerical; Maxwell–Fourier method

1. Introduction

1.1. Context of This Paper

Political and economic issues are one of the main drawback of permanent-magnet (PM) synchronous machines (PMSMs) due to the presence of rare-earth PMs. Indeed, economic dependency constitutes a strong objective for industrial electronics companies and that is the reason why industry and academia conduct research on PM-less machines (e.g., synchronous or switched-reluctance machines, induction machines) [1]. However, today, PMSMs are one of the most competitive machines for their high electromagnetic performances, massive torque, high efficiency, and low torque ripple [2,3]. Nevertheless, the speed variation leads to variable magnetic fields constituted of: (i) temporal harmonics due to the current waveform (e.g., sinusoidal, six-step rectangular, pulse-width modulation currents, etc.), and (ii) spatial harmonics, both the stator slotting permeance and the magnetomotive force (MMF) distribution [4,5]. Consequently, eddy-currents appear inside the PM volume, which contributes to supplement losses, namely eddy-current losses.

At high-speed or high-frequency, PM losses can be important [4]. The electrical conductivity can also be affected at high temperatures, leading to a loss increase and faulty conditions due to the PM. Therefore, the study of this phenomenon is required to predict the PM eddy-current losses in order to improve the design procedure in electromagnetic devices. Different formulations have been developed in order to estimate these eddy-current losses, such as: (i) semi-analytical methods based on the

electrical equivalent circuit (EEC) and/or magnetic equivalent circuit (MEC) [6], (ii) analytical methods based on the formal resolution of Maxwell's equations [4,7–10], and (iii) the numerical hybrid method based on the 3-D finite-element analysis (FEA) and the 3-D finite-difference method [11]. In [12,13], the model enables consideration of both spatial and temporal harmonics using a resistance-limited magnetic potential vector to formulate the system resolution. In [14], the eddy-currents induced by the magnetic field variation are computed by solving the Maxwell's equations. The losses are calculated by using the integral volume accordingly with the eddy-current calculated previously. Also, eddy-currents could be considered as additional induced terms in the Ampere laws [15]. The armature reaction can be modeled by MMF sources [16], or by including hysteresis and eddy-current coefficients [17,18].

In [19,20], the eddy-currents are obtained by additional capacitors in the MEC. Coupling an EEC with a MEC or a 2-D solution of Maxwell's equations is also used in [21]. The authors conclude that the armature reaction does not contribute significantly to the increase of the PM eddy-current losses. The advantage of this method consists of the separation of the magnetic phenomenon from those of the EEC responsible for the magnetic field reaction. In [22], a magnetic inductance can also be incorporated in the MEC where the eddy-currents can be modeled by short single coil encircling iron elements with a resistance. In [23,24], an EEC taking into account to the magnetic field reaction has been considered. The local quantities can be derived from MEC or FEA. Using the EEC, the authors consider the eddy-currents by incorporating a magnetic inductance in a regular MEC. This was applied to a static electromagnetic device as well as a PMSM. A similar approach based on a coupling model between a MEC and an EEC is used in [25], where the 3-D MEC serves to compute the flux crossing perpendicularly the section of the massive conductive part and gives the induced voltage, while the EEC is used to compute the eddy-currents losses [26]. It is explained in [27] how to incorporate the eddy-current losses in 3-D FEA. It can also be found in [28] a 3-D approach method using a magnetic conductance. An eddy-current loss estimation made by using an EEC is detailed in [29] to modelize a PMSM, where the resolution of equation systems are done simultaneously with multi-slice 2-D FEA. A hybrid method for the eddy-current loss calculation was proposed in [30]. It combines a 2-D transient FEA to obtain the magnetic field distribution in PMs and a 2-D analytical model to determine resulting eddy-current losses. The FEA output is used as the data input of the analytical method. The results are validated by 3-D transient FEA computations and by experimental measurements.

1.2. Objectives of This Paper

The major drawbacks of the previously cited papers are linked to the high computational time and depend strongly on the FEA. A model coupling (or a hybrid model), combining an analytical model based on the Maxwell–Fourier method (i.e., the formal resolution of Maxwell's equations by using the separation of variables method and the Fourier's series) in massive conductive parts (e.g., aluminum, PMs, copper, iron) with a generic MEC, appeared as a promising solution [31]. MECs are largely used in modeling with a greater or lesser time depending on the fineness applied to the reluctances network. Analytical methods are also well-known for their short computation time. Therefore, unlike [30], it is necessary to combine these two models in order to find a good compromise between the computation time and the model accuracy. Hence, the scientific objective of this paper is to describe this type of hybrid model by validating it with numerical and experimental results.

The 2-D generic MEC determine the magnetic flux density distribution in massive conductive parts without the skin effect (i.e., without the eddy-current reaction field). The 2-D analytical model based on the Maxwell–Fourier method calculates the magnetic field distribution in massive conductive parts considering the skin effect as well as the resultant eddy-current density. The boundary conditions (BCs) imposed on the 2-D analytical model are equivalent to the magnetic field obtained from the MEC. Therefore, the 2-D analytical model will be applied across different layers in the y-axis of massive conductive parts. From local quantities with the skin effect, the 3-D eddy-current loss distribution in massive conductive parts can be observed. It is interesting to note that special attention should be paid to BCs of the 2-D analytical model. Frequently, only the middle component of the magnetic field (i.e.,

by assuming a uniform magnetic field) is taken to calculate the eddy-current losses [7,30]. In this work, the BC number influence is accounted for by applying three different limit conditions. Moreover, this paper contributes to the study of the discretization influence in order to reduce the computer time consumed for optimization of the design and takes into account to the eddy-current losses in thermal design in a more accurate manner.

This paper is organized as follows. First, Section 2 describes the U-cored static electromagnetic device used to validate the proposed approach with 3-D FEA and experimental results. Secondly, the model coupling is exposed by describing the 2-D generic MEC and the 2-D analytical model based on the Maxwell–Fourier method. The magnetic field distribution for various models is validated with 2-D and 3-D FEA [32]. The mathematic formulation as well as the experimental and 3-D numerical validation of eddy-current losses in massive conductive parts in aluminum are given in Section 3. The discretization influence is also discussed in the same section.

2. Model Coupling: 2-D Generic MEC/Maxwell–Fourier

2.1. U-Cored Static Electromagnetic Device

The 2-D view of the U-cored static electromagnetic device is shown in Figure 1. It is constituted of a mobile armature that allows the insertion of massive aluminum conductive parts of various thicknesses. Two coils having N_t series turns are connected in parallel. The electromagnetic device is supplied with a sinusoidal voltage. The magnetic circuit is not saturated with the voltage levels. Therefore, the current waveform is purely sinusoidal with a maximum amplitude of $I_{\max}$. The current direction in the conductor is defined by $\otimes$ for the forward conductor and $\odot$ for return conductor. The U-cored static electromagnetic device as well as the experimental tests have been presented in [33]. The geometrical and physical parameters are detailed respectively in Tables 1 and 2.

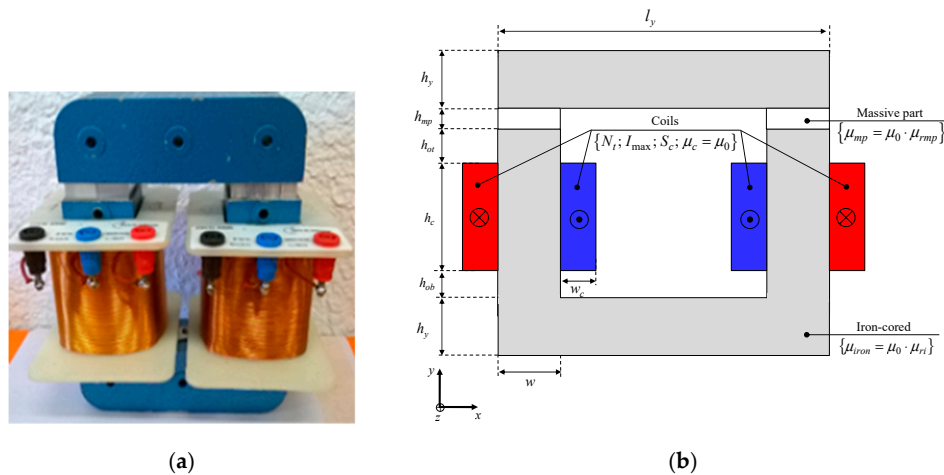


Figure 1. U-cored static electromagnetic device: (a) Experimental test [33], and (b) geometrical parameters (see Table 1 for the various parameters).

Table 1. Geometrical parameters.

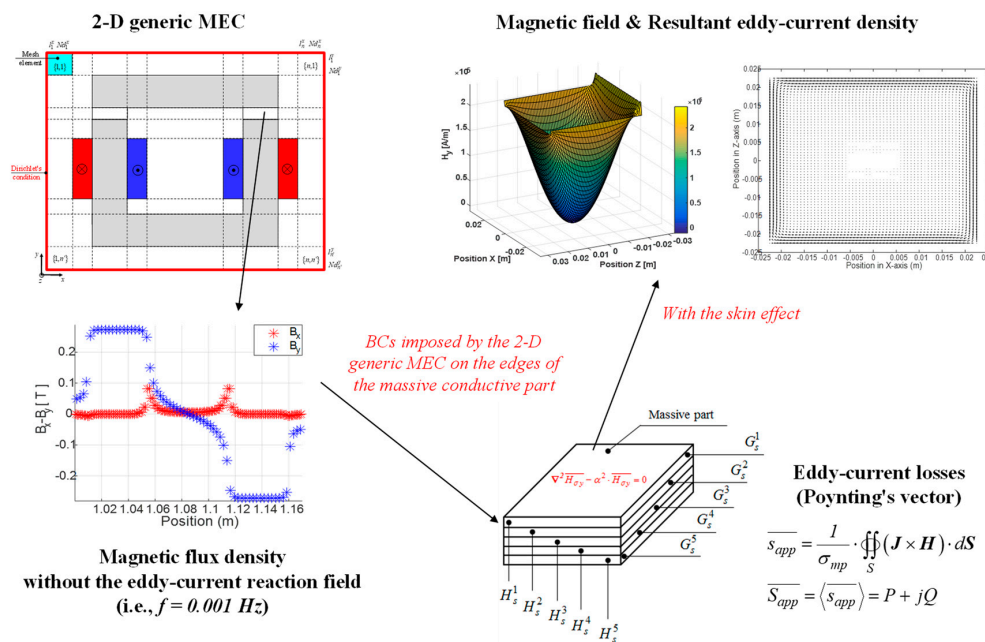
Parameters, Symbols (Units)	Values
Depth, d (mm)	43
Width, w (mm)	43
Coil height and width, $\{h_c; w_c\}$ (mm)	{77; 10}
Coil section, $S_c = h_c \cdot w_c$ (mm ²)	770
Yoke height and length, $\{h_y; l_y\}$ (mm)	{43; 150}
Thickness of massive part, h_{mp} (mm)	6 or 10
Height of overhang top and low, $\{h_{ot}; h_{ob}\}$ (mm)	{19; 4}

Table 2. Physical parameters.

Parameters, Symbols (Units)	Values
Electrical frequency, f (Hz)	50
Maximal current, $I_{\max}$ (A)	0 to 8.2
Number of turns, N_t (-)	500
Relative permeability of massive parts in aluminum, μ_{mp} (-)	1
Electrical conductivity of massive parts in aluminum, σ_{mp} (S/m)	38.46×10^6
Vacuum permeability, μ_0 (H/m)	$4\pi \times 10^{-7}$
Relative permeability of iron core, μ_n (-)	1500

2.2. Proposed Approach

The approach consists of combining two models, viz.: (i) a 2-D generic MEC, and (ii) a 2-D analytical model based on the Maxwell–Fourier method (i.e., the formal resolution of Maxwell’s equations by using the separation of variables method and the Fourier’s series). The 2-D generic MEC gives us the ability to determine the magnetic flux density distribution in massive conductive parts without the skin effect, while the 2-D analytical model provides the local quantities with the skin effect. By applying the Poynting vector, the eddy-current losses in massive conductive parts across a closed surface can be determined. Figure 2 shows the principle of model coupling.

**Figure 2.** Principle of model coupling.

It can be noticed that the input data of the 2-D generic MEC are the geometrical and physical parameters (see Tables 1 and 2) as well as the discretization vectors in both directions (i.e., x- and y-axis). The output data of the 2-D generic MEC (i.e., the magnetic flux density without the skin effect) will be used as the input data of the 2-D analytical model, where the main variables are the skin depth of the massive conductive part and the spatial harmonics number.

2.3. 2-D Generic MEC

2.3.1. General Assumptions

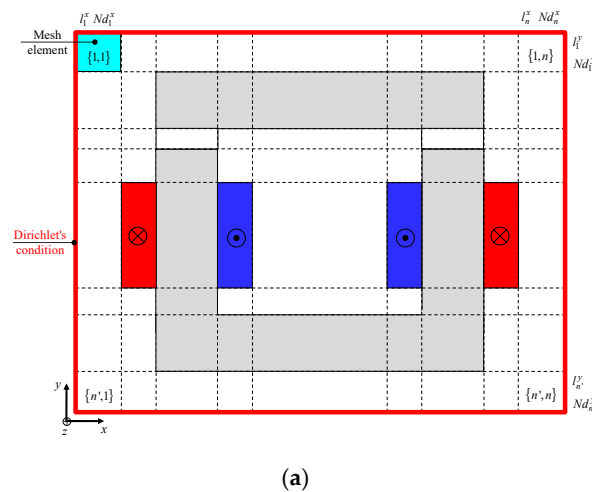
The 2-D generic MEC is based on the following simplifying assumptions:

- The saturation and hysteresis effects are neglected;

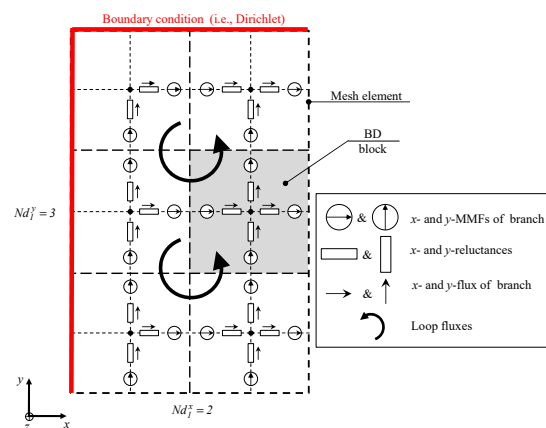
- The end-effects in the z-axis are neglected (i.e., the semi-analytical is assumed to be in 2-D);
- The eddy-current effects in all materials (e.g., the massive parts, the copper, the iron) are neglected (i.e., the electrical conductivities are assumed to be null);
- The magnetic materials are considered as isotropic;
- The mechanical stress on the nonlinear $B(H)$ curve is ignored;
- Since the magnetic circuit is not saturated, the magnetic permeability is supposedly constant, corresponding to the linear zone of the nonlinear $B(H)$ curve.

2.3.2. Automatic Mesh

In an (x, y) coordinate system, Figure 3 represents the generalized discretization of a U-cored static electromagnetic device for the development of 2-D generic MEC [34,35]. The device is inserted in an infinite box whose outer edges respect the Dirichlet's conditions. It can be divided into $n = 9$ zones in the x-axis and $n' = 8$ zones in the y-axis. The intersection of these zones in both axes gives rise to mesh elements $\{j, i\}$, having the same magnetic permeability, of size $l_i^x \times l_j^y$ with $i = 1, \dots, n$ and $j = 1, \dots, n'$. So, the total number of mesh elements is equal to $n \times n' = 72$. The mesh elements $\{j, i\}$ can be discretized one or several bidirectional (BD) blocks from $\{Nd_j^y, Nd_i^x\}$ which are respectively the vectors (of dimension $n' \times 1$ and $n \times 1$) of discretization number in the y- and x-axis for the zone j and i (see Figure 3a).



(a)



(b)

Figure 3. Generalized discretization of a U-cored static electromagnetic device: (a) Automatic mesh, and (b) discretization of a mesh element (e.g., for the mesh element $\{1,1\}$).

The mesh elements are so composed of BD blocks depending on the discretization chosen by the designer. Figure 3b describes an example of discretization for the mesh element $\{1, 1\}$ (see the mesh element in sky blue color in Figure 3a) with $Nd_1^x = 2$ and $Nd_1^y = 3$, where the number of BD blocks is equal to $Nd_1^x \times Nd_1^y = 6$. The BD blocks, connected between them by the loop fluxes ψ and giving to the magnetic flux the possibility to flow in both directions, are described by a middle-point related to (except for the outer edges of the device due to Dirichlet's conditions):

- 4 branch MMFs (i.e., two x-MMFs and two y-MMFs);
- and 4 magnetic reluctances (i.e., two x-reluctances and two y-reluctances) crossed by branch fluxes φ .

In general, the number of loop fluxes ψ is given by

$$N_\psi = N_\psi^x \cdot N_\psi^y, \quad (1a)$$

$$N_\psi^x = p - 1 \text{ with } p = \sum_{i=1}^n Nd_i^x, \quad (1b)$$

$$N_\psi^y = m - 1 \text{ with } m = \sum_{j=1}^{n'} Nd_j^y. \quad (1c)$$

The number of magnetic reluctances (or branch fluxes and MMFs) is defined by

$$N = N^x + N^y \quad (2a)$$

$$N^x = m \cdot P \text{ with } P = 2N_\psi^x, \quad (2b)$$

$$N^y = p \cdot M \text{ with } M = 2N_\psi^y. \quad (2c)$$

It is interesting to note that the number of BD blocks can be given by $N_{BD} = p \cdot m$.

One should notice that the accuracy and the computational time of 2-D generic MEC rise by increasing the number of BD blocks in each mesh element.

2.3.3. Matrix Formulation

Using the Maxwell's equations as well as the magnetic material equations, the 2-D generic MEC (where the loop fluxes ψ are the unknowns) can be governed by

$$[F] - [\chi] \cdot [\mathfrak{R}] \cdot [\chi]^T \cdot [\psi] = 0, \quad (3a)$$

$$[F] = [\chi] \cdot [MMF], \quad (3b)$$

in which

- $[\psi]$ is the loop fluxes vector (of dimension $N_\psi \times 1$);
- $[F]$ is the loop MMFs vector (of dimension $N_\psi \times 1$);
- $[MMF]$ is the branch MMFs vector (of dimension $N \times 1$) defined by

$$[MMF] = \begin{bmatrix} [MMF^x] \\ [MMF^y] \end{bmatrix}. \quad (4)$$

The branch MMFs vectors $[MMF^x]$ and $[MMF^y]$ in the x- and y-axis (of dimension $N^x \times 1$ and $N^y \times 1$) are given by

$$[MMF^\bullet] = \begin{bmatrix} MMF_{\{1,1\}}^\bullet \\ \vdots \\ MMF_{\{1,n\}}^\bullet \\ \vdots \\ MMF_{\{n',1\}}^\bullet \\ \vdots \\ MMF_{\{n',n\}}^\bullet \end{bmatrix}, \quad (5)$$

with

$$MMF_{\{j,i\}}^x = \begin{cases} [Z]_{(2 \cdot Nd_i^x - 1) \cdot Nd_j^y, 1} \text{ for } i = \begin{vmatrix} 1 \\ n \end{vmatrix} & \forall j \\ \text{otherwise } [Z]_{2 \cdot Nd_i^x \cdot Nd_j^y, 1} \end{cases} \quad (6a)$$

$$MMF_{\{j,i\}}^y = \begin{cases} [Z]_{(2 \cdot Nd_j^y - 1) \cdot Nd_i^x, 1} \text{ for } j = \begin{vmatrix} 1 \\ n' \end{vmatrix} & \forall i \\ f_{MMF}(N_t, I_{\max}) \cdot [O]_{2 \cdot Nd_i^x \cdot Nd_j^y, 1} \text{ for } \begin{vmatrix} j = 5 \\ i = 2 \text{ to } 4 \wedge 6 \text{ to } 8 \end{vmatrix} \\ \text{otherwise } [Z]_{2 \cdot Nd_i^x \cdot Nd_j^y, 1} \end{cases} \quad (6b)$$

where $[Z]_{\bullet, \bullet}$ is the zeros matrix of dimension $\bullet \times \bullet$, $[O]_{\bullet, \bullet}$ is the ones matrix of dimension $\bullet \times \bullet$, and $f_{MMF}(N_t, I_{\max})$ is a MMF function explained in [35]. Figure 4a represents the waveform of this function at $t = 0$ s corresponding to $I_{\max}$. The MMF curve for a coil is defined by a trapezoidal waveform whose Ampere-turns change linearly from 0 to $N_t \cdot I_{\max}$ for the forward conductor $\otimes$ and from $N_t \cdot I_{\max}$ to 0 for the return conductor $\odot$. Figure 4b describes an example of MMF values according to the discretization number for the mesh element with $Nd_5^y = 2$ and $Nd_2^x = 2$. All BD blocks in the y-axis have the same values of MMFs. The MMF values differ with the discretization number according to the MMF slope in the conductor.

- $[\mathfrak{R}]$ the diagonal matrix of magnetic reluctances (of dimension $N \times N$) defined by

$$[\mathfrak{R}] = \begin{bmatrix} [\mathfrak{R}^x] & 0 \\ 0 & [\mathfrak{R}^y] \end{bmatrix}, \quad (7)$$

The diagonal matrices of magnetic reluctances $[\mathfrak{R}^x]$ and $[\mathfrak{R}^y]$ in the x- and y-axis (of dimension $N^x \times N^x$ and $N^y \times N^y$) are given by

$$[\mathfrak{R}] = \begin{bmatrix} [\mathfrak{R}^x] & 0 \\ 0 & [\mathfrak{R}^y] \end{bmatrix}, \quad (8)$$

with

$$[\mathfrak{R}_{\{j,i\}}^x] = \mathfrak{R}_{j,i}^x \cdot \begin{cases} [O]_{Nd_j^y, 2 \cdot Nd_i^x - 1} \text{ for } i = \begin{vmatrix} 1 \\ n \end{vmatrix} & \forall j \\ \text{otherwise } [O]_{Nd_j^y, 2 \cdot Nd_i^x} \end{cases} \quad (9a)$$

$$[\mathfrak{R}_{\{j,i\}}^y] = \mathfrak{R}_{j,i}^y \cdot \begin{cases} [O]_{2 \cdot Nd_j^y - 1, Nd_i^x} \text{ for } j = \begin{vmatrix} 1 \\ n' \end{vmatrix} & \forall i \\ \text{otherwise } [O]_{2 \cdot Nd_j^y, Nd_i^x} \end{cases}, \quad (9b)$$

where

$$\mathfrak{R}_{j,i}^x = L_i^x / (\mu_{j,i} \cdot S_j^x), \quad (10a)$$

$$\mathfrak{R}_{j,i}^y = L_i^y / (\mu_{j,i} \cdot S_j^y), \quad (10b)$$

in which $\mu_{j,i}$ is the absolute magnetic permeability of mesh elements $\{j, i\}$ defined by

$$\mu_{j,i} = \mu_0 \cdot \begin{cases} \mu_{ri} & \text{for } \begin{cases} \{2 \wedge 7, 3 \text{ to } 7\} \\ \{4 \text{ to } 6, 3 \wedge 7\} \end{cases} \\ \mu_{rmp} & \text{for } \{3, 3 \wedge 7\} \\ \text{otherwise } 1 & \end{cases} \quad (11)$$

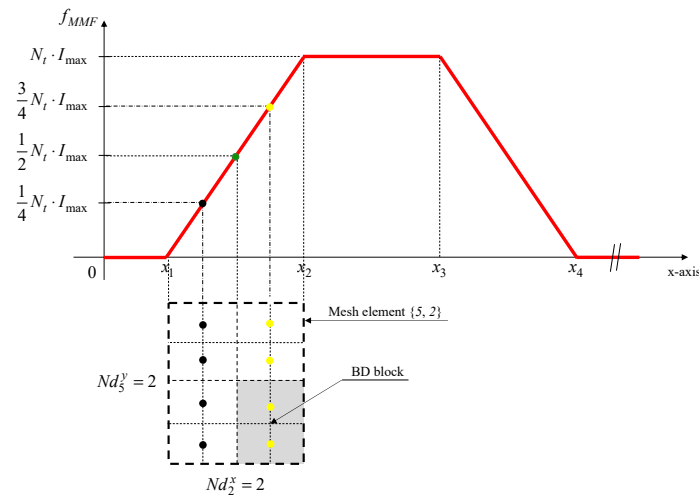
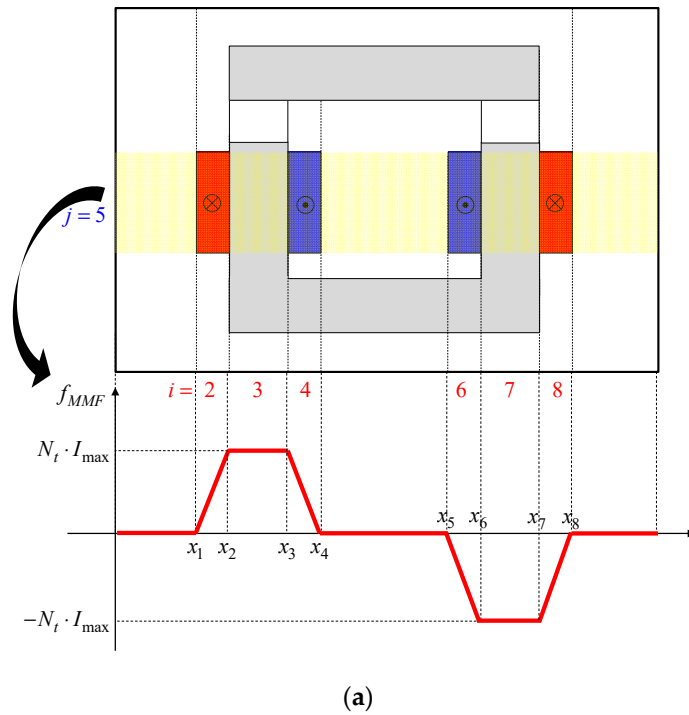


Figure 4. The magnetomotive force (MMF) function $f_{MMF}(N_t, I_{\max})$: (a) Waveform, and (b) MMF values according to the discretization number (e.g., for the mesh element $\{5, 2\}$ with $Nd_5^y = 2$ and $Nd_2^x = 2$).

The lengths (viz., L_i^x and L_j^y) and sections (viz., S_j^x and S_i^y) of magnetic reluctances in the x- and y-axis are given in Table 3.

Table 3. Lengths and sections of magnetic reluctances.

	Length (m)	Section (m ²)
x-axis	$L_i^x = \frac{l_i^x}{2 \cdot Nd_i^x}$	$S_j^x = \frac{d \cdot l_j^y}{Nd_j^y}$
y-axis	$L_j^y = \frac{l_j^y}{2 \cdot Nd_j^y}$	$S_i^y = \frac{d \cdot l_i^x}{Nd_i^x}$

- $[\chi]$ is the topological (or incidence) matrix (of dimension $N_\psi \times N$) defined by

$$[\chi] = \begin{bmatrix} [\chi^x] & [\chi^y] \end{bmatrix}, \quad (12)$$

where $[\chi^x]$ and $[\chi^y]$ are respectively the topological matrices in the x- and y-axis (of dimension $N_\psi \times N^x$ and $N_\psi \times N^y$). The elements $[\chi]_{k,k'}$ are then equal to [36]:

$$[\chi]_{k,k'} = \begin{cases} \pm 1 & \text{if } \phi_{k'} \in \psi_k^\pm \\ 0 & \text{if } \phi_{k'} \notin (\psi_k^+ \cup \psi_k^-) \end{cases}, \quad (13)$$

with ψ_k^+ —branch and loop fluxes have the same direction, and —branch and loop fluxes have opposite directions. Therefore, the topological matrices $[\chi^x]$ and $[\chi^y]$ are given by

$$[\chi^x] = [Y]_{N_\psi^y} \otimes ([I]_{N_\psi^x, N_\psi^x} \otimes [O]_{1,2}), \quad (14a)$$

$$[\chi^y] = [I]_{N_\psi^y, N_\psi^y} \otimes \begin{bmatrix} [Y]_{N_\psi^x} & [Y]_{N_\psi^x} \end{bmatrix}, \quad (14b)$$

where $[I]_{*,*}$ is the identity matrix of dimension $* \times *$, $\otimes$ is the Kronecker's product, and

$$[Y]_* = \begin{bmatrix} 1 & 2 & \cdots & \cdots & *+1 \\ -1 & 1 & & & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ \vdots \\ * \end{matrix}. \quad (15)$$

2.3.4. Problem Solving

To solve the Cramer's system (3), a numerical matrix inversion is required for the calculation of $[\psi]$, viz., $[\psi] = [A]^{-1}[F]$ with $[A] = [\chi] \cdot [\mathfrak{R}] \cdot [\chi]^T$. For a saturated system, it is interesting to note that (3) can be solved iteratively with a constant relative magnetic permeability μ_{ri} according to the nonlinear $B(H)$ curve at each iteration by using the fixed-point iteration method. The flowchart of the nonlinear system solving is detailed in [37].

Knowing $[\psi]$, the branch fluxes vector $[\phi]$ (of dimension $N \times 1$) and the magnetic flux densities vector $[B]$ (of dimension $N \times 1$) are respectively defined by

$$[\phi] = [\chi]^T \cdot [\psi], \quad (16)$$

$$[B] = [\phi]/[S], \quad (17)$$

with the reluctances surface vector (of dimension $N \times 1$) in the various BD blocks given by

$$[S] = \begin{bmatrix} [S^x] \\ [S^y] \end{bmatrix} \quad (18)$$

The reluctances surface vectors $[S^x]$ and $[S^y]$ in the x- and y-axis (of dimension $N^x \times 1$ and $N^y \times 1$) are given by

$$[S^\bullet] = \begin{bmatrix} S_{\{1,1\}}^\bullet \\ \vdots \\ S_{\{1,n\}}^\bullet \\ \vdots \\ S_{\{n',1\}}^\bullet \\ \vdots \\ S_{\{n',n\}}^\bullet \end{bmatrix}, \quad (19)$$

with

$$S_{\{j,i\}}^x = S_{j,i}^x \cdot \begin{cases} [Z]_{(2 \cdot Nd_i^x - 1) \cdot Nd_j^y, 1} & \text{for } i = \begin{vmatrix} 1 \\ n \end{vmatrix} \\ \text{otherwise } [Z]_{2 \cdot Nd_i^x \cdot Nd_j^y, 1} \end{cases} \quad \forall j \quad (20a)$$

$$S_{\{j,i\}}^y = S_{j,i}^y \cdot \begin{cases} [Z]_{(2 \cdot Nd_j^y - 1) \cdot Nd_i^x, 1} & \text{for } j = \begin{vmatrix} 1 \\ n' \end{vmatrix} \\ \text{otherwise } [Z]_{2 \cdot Nd_i^x \cdot Nd_j^y, 1} \end{cases} \quad \forall i \quad (20b)$$

2.3.5. Comparing with 2-D FEA

The validation of 2-D generic MEC has been realized by Cedrat's Flux2D software package (i.e., an advanced FE method based numeric field analysis program) [32]. The parameters of a U-cored static electromagnetic device have been sent to a 2-D FEA pre-processor in the application "Magneto Static 2-D". The 2-D FEA is done with the same assumptions as the 2-D generic MEC (see Section 2.3.1). It has been implemented in Matlab® by using the sparse matrix/vectors. The discretization in the x- and y-axis have been considered as follows

$$Nd^x = \begin{bmatrix} 1 & 4 & k & 4 & 14 & 4 & k & 4 & 1 \end{bmatrix}, \quad (21a)$$

$$Nd^y = \begin{bmatrix} 1 & 10 & k & 5 & 6 & 6 & 10 & 1 \end{bmatrix}, \quad (21b)$$

where $k = 2; 6; 10; 14; 18; 22; 24$ is the discretization number in massive conductive parts.

Consequently, for the high discretization (i.e., $k = 24$), (3) is composed of $N_{BD} = 5040$ BD blocks, $N_\psi = 4898$ loop fluxes, and $N = 19,874$ branch fluxes, which is much smaller than the 2-D FEA mesh having 38,897 nodes, 2081 line elements, and 19,288 surface elements of the second order (viz., the triangles number of system). Figure 5 shows the consumption time for the 2-D generic MEC versus k . By using the high discretization (i.e., $k = 24$) in the 2-D generic MEC, the computation time is the same for both modeling methods, viz., ~4 s.

The validation paths of $\mathbf{B} = \{B_x; B_y; 0\}$ for the comparison are given in Figure 6. The waveforms of B_x and B_y are represented on various paths in Figures 7–9 for $I_{\max} = 7.78$ A at $t = 0$ s and $h_{mp} = 6$ mm. The dotted lines represent the components of $\mathbf{B}$ calculated by the 2-D FEA and the circles correspond to 2-D generic MEC. It can be seen that a very good agreement is obtained for the components of $\mathbf{B}$ whatever the paths. Figures 7a and 9b confirm that the electromagnetic device is not saturated with a maximum level of $\mathbf{B}$ equal to 1 T. In Figure 8a, it is interesting to note that the level of B_x on the edges of massive conductive parts are not the same, which is due to the electromagnetic device structure. Indeed, the magnetic leakages are more important inside than outside. It should be noted that B_x in massive conductive parts is considered negligible in relation to B_y (see Figure 8). Moreover, Figure 10 presents a zoom of B_y in the left massive conductive part for the various paths (viz., Path_{mp1} to Path_{mp4}) between x_1 and x_4 (see Figure 6). Due to leakage fluxes, the levels of B_y are different at the edges and in

the middle of the massive conductive part whatever the path in the y-axis. These various magnetic flux densities will be used as BCs in the 2-D analytical model based on the Maxwell–Fourier method, which calculate the magnetic field distribution considering the skin effect as well as the resultant eddy-current density. It is interesting to note that the path number depends on the discretization number in the y-axis, so the influence of the discretization number will be discussed in Section 3 in the eddy-current loss calculation.

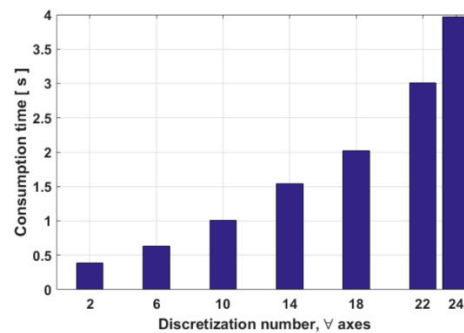


Figure 5. Consumption time for the 2-D generic magnetic equivalent circuit (MEC) according to k (viz., the discretization number in massive conductive parts).

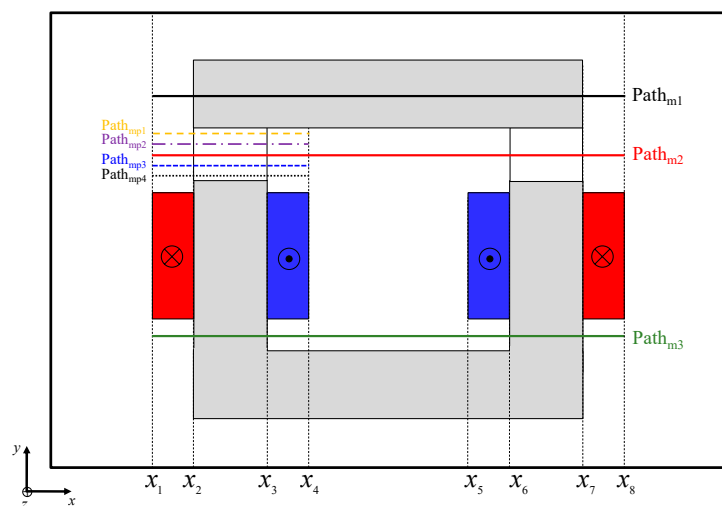


Figure 6. Paths of magnetic flux density validation for the comparison.

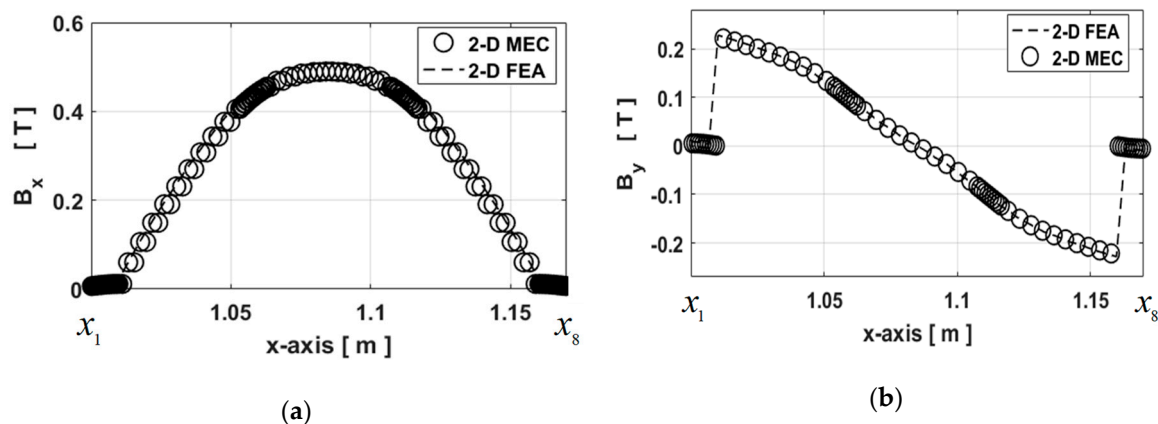


Figure 7. Waveform of B for Path_{m1} with $I_{\max} = 7.78\text{A}$ at $t = 0\text{s}$ and $h_{mp} = 6\text{mm}$: (a) x- and (b) y-component.

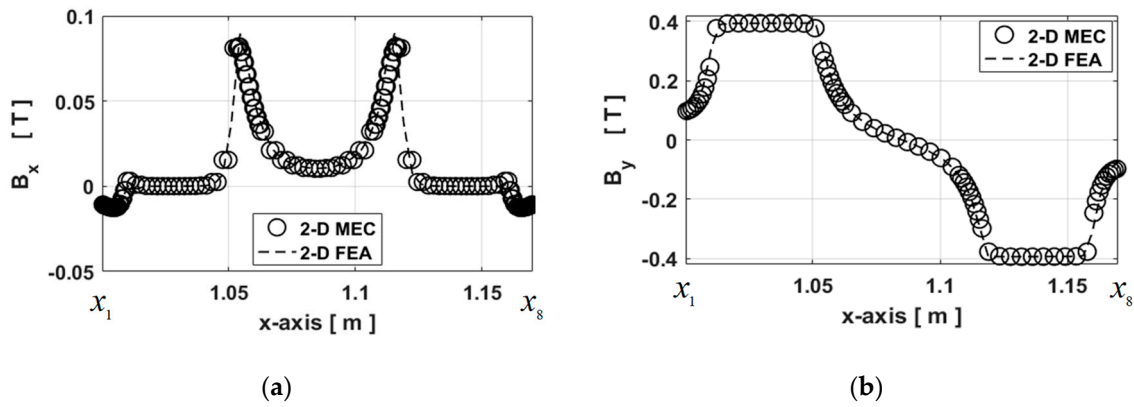


Figure 8. Waveform of B for Path_{m2} with $I_{\max} = 7.78$ A at $t = 0$ s and $h_{mp} = 6$ mm: (a) x- and (b) y-component.

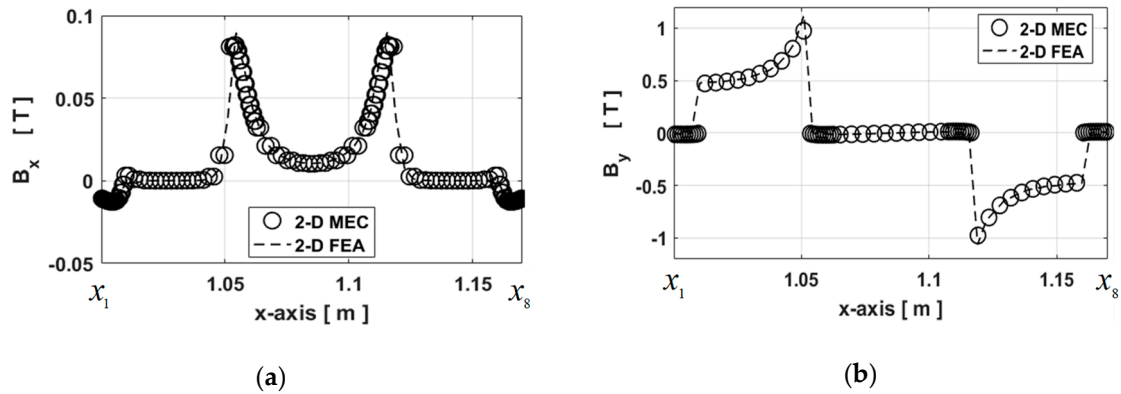


Figure 9. Waveform of B for Path_{m3} with $I_{\max} = 7.78$ A at $t = 0$ s and $h_{mp} = 6$ mm: (a) x- and (b) y-component.

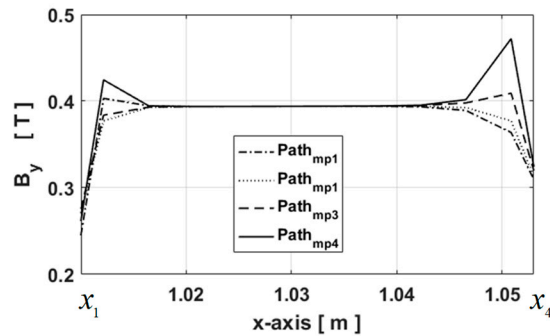


Figure 10. The y-component of B in the left massive conductive part (between x_1 and x_4) for various paths with $I_{\max} = 7.78$ A at $t = 0$ s and $h_{mp} = 6$ mm.

2.4. 2-D Maxwell–Fourier

2.4.1. General Assumptions

The 2-D analytical model based on the Maxwell–Fourier method is defined by the following simplifying assumptions:

- The massive conductive parts are excited by the magnetostatic magnetic field from the 2-D generic MEC which is assumed normal to the xz -plane;
- Since the magnetic circuit is not saturated (see Figures 7a and 9b), the excitation magnetic field varies sinusoidally in time which is similar to the power supply source;

- The resultant eddy-current density in massive conductive parts has two components, i.e., $J = \{J_x; 0; J_z\}$;
- The relative magnetic permeability and electrical conductivity of massive conductive parts (i.e., μ_{mp} and σ_{mp}) are assumed to be constant.

It is interesting to note that the 2-D analytical model, for calculating the magnetic field distribution with the skin effect as well as the resultant eddy-current density, will be applied across different layers in the y -axis of massive conductive parts (viz., the mesh elements $\{3, 3\}$ and $\{3, 7\}$ in Figure 3a). These different layers depend on the discretization Nd_3^y of BD blocks in massive conductive parts.

2.4.2. Governing Partial Differential Equations (PDEs) in Cartesian Coordinates

By assuming that the term $\partial D / \partial t$ (with D as the displacement field vector) is negligible in comparison with the resultant eddy-current density J , the Maxwell's equations are represented by

$$\nabla \times H = J \text{ implying } \nabla \cdot J = 0 \text{ (Maxwell – Ampère)}, \quad (22a)$$

$$\nabla \times E = -\partial B / \partial t \text{ (Maxwell – Faraday)}, \quad (22b)$$

$$\nabla \cdot B = 0 \text{ (Maxwell – Thomson)}, \quad (22c)$$

where E is the electrical field vector.

In a conductor, E is linked to J by

$$J = \sigma \cdot E \text{ (Ohm's law)}, \quad (23)$$

where σ is the electrical conductivity.

The field vectors B and H are coupled by

$$B = \mu \cdot H + \mu_0 \cdot M_r \text{ (Magnetic material equation)}, \quad (24)$$

where M_r is the remnant magnetization vector (with $M_r \neq 0$ for the PMs or $M_r = 0$ for the other materials).

Inside a linear magnetic or nonmagnetic material of constant electrical conductivity without electromagnetic sources (i.e., $M_r = 0$), the magnetodynamic PDEs in terms of H can be defined by

$$\nabla^2 H - \mu \cdot \sigma \cdot \frac{\partial H}{\partial t} = 0 \text{ (Diffusion equation)}. \quad (25)$$

From the general assumptions, and using the complex notation, the magnetic field $H = \{0; H_{\sigma y}; 0\}$ inside the massive conductive part considering the skin effect can be written as

$$H_{\sigma y} = \Re \{ \overline{H_{\sigma y}} \cdot e^{j \cdot \omega \cdot t} \} \quad (26)$$

where $j = \sqrt{-1}$ and $\omega = 2\pi \cdot f$ is the electrical pulse.

Therefore, (25) becomes

$$\nabla^2 \overline{H_{\sigma y}} - \alpha^2 \cdot \overline{H_{\sigma y}} = 0 \text{ with } \alpha^2 = j \cdot \mu_{mp} \cdot \sigma_{mp} \cdot \omega, \quad (27)$$

which is the complex Helmholtz's equation.

In (x, z) coordinate system, the distribution of the magnetic field inside the massive conductive part considering the skin effect is then governed by

$$\frac{\partial^2 \overline{H_{\sigma y}}}{\partial x^2} + \frac{\partial^2 \overline{H_{\sigma y}}}{\partial z^2} - \alpha^2 \cdot \overline{H_{\sigma y}} = 0 \quad (28)$$

2.4.3. Definition of BCs

The BCs at the edges of massive conductive parts for the 2-D analytical model are equivalent to the magnetostatic magnetic fields of 2-D generic MEC (see Section 2.3). Usually, BCs are considered homogeneous at the edges, which are often equal to the excitation magnetic field value in the middle of the massive conductive part as [7,30]. From the 2-D generic MEC simulations, the magnetic field levels are different at the edges and in the middle of the massive conductive part, whatever the path in the y-axis (see Figure 10). Hence, BCs in the 2-D analytical model are considered as non-homogeneous. Figure 11 represents the BCs at the edges of massive conductive parts in a (x, z) coordinate system and $\forall l$, where M_s^l , L_s^l , and R_s^l are respectively the magnetostatic magnetic field values in the middle, at the left edge, and at the right edge of massive conductive parts. The index $l = 1, \dots, 2 \cdot Nd_3^y$ is the path in the y-axis (or the parallel path in the x-axis). Figure 12 shows the value locations of M_s^l , L_s^l , and R_s^l in the massive conductive part from the 2-D generic MEC in a (x, y) coordinate system.

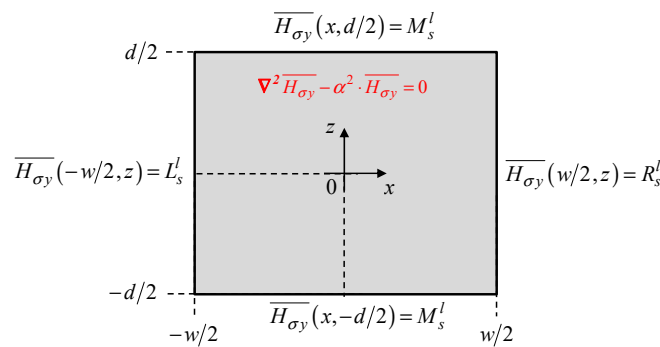


Figure 11. Boundary conditions (BCs) at edges of massive conductive parts in a (x, z) coordinate system and $\forall l$.

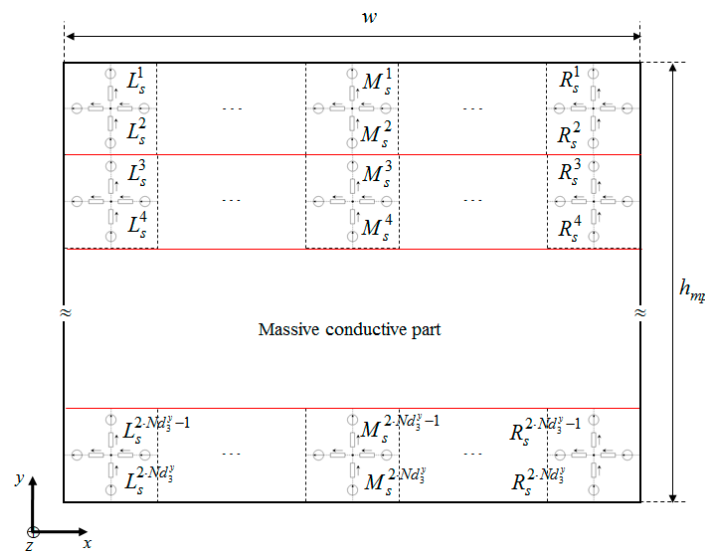


Figure 12. The values location of M_s^l , L_s^l , and R_s^l in the massive conductive part from the 2-D generic MEC in a (x, y) coordinate system.

2.4.4. General Solution of the Magnetic Field

Using the separation of variables method, the 2-D general solution of $H_{\sigma y}$ in both directions (i.e., x- and y-edges) can be written as a Fourier's series, $\forall l$,

$$\overline{H_{\sigma y}} = \sum_{h=0}^{\infty} \left[\begin{array}{c} \overline{c_{xh}^l} \cdot \text{ch}(\chi_h \cdot z) \\ \dots + \overline{d_{xh}^l} \cdot \text{sh}(\chi_h \cdot z) \end{array} \right] \cdot \left[\begin{array}{c} \overline{e_{xh}^l} \cdot \cos(\beta_h \cdot x) \\ \dots + \overline{f_{xh}^l} \cdot \sin(\beta_h \cdot x) \end{array} \right] + \sum_{k=0}^{\infty} \left[\begin{array}{c} \overline{c_{zk}^l} \cdot \cos(\lambda_k \cdot z) \\ \dots + \overline{d_{zk}^l} \cdot \sin(\lambda_k \cdot z) \end{array} \right] \cdot \left[\begin{array}{c} \overline{e_{zk}^l} \cdot \text{ch}(\delta_k \cdot x) \\ \dots + \overline{f_{zk}^l} \cdot \text{sh}(\delta_k \cdot x) \end{array} \right], \quad (29a)$$

where $\overline{c_{xh}^l} \sim \overline{f_{xh}^l}$ and $\overline{c_{zk}^l} \sim \overline{f_{zk}^l}$ are the integration constants, β_h and λ_k are the periodicity of $\overline{H_{\sigma y}}$ in the x- and z-axis, h and k are the spatial harmonic orders, and

$$\chi_h = \sqrt{\alpha^2 + \beta_h^2}, \quad (29b)$$

$$\delta_k = \sqrt{\alpha^2 + \lambda_k^2}. \quad (29c)$$

The coefficients $\overline{c_{xh}^l} \sim \overline{f_{xh}^l}$ and $\overline{c_{zk}^l} \sim \overline{f_{zk}^l}$ are determined by applying the BCs illustrated in Figure 9. Therefore, (29a) becomes, $\forall l$,

$$\overline{H_{\sigma y}} = H_s^l \cdot \left\{ \frac{\text{ch}(\alpha \cdot z)}{\text{ch}(\alpha \cdot \frac{d}{2})} + \sum_{k=1,3,\dots}^{\infty} \left[\overline{e_{zk}^l} \cdot \frac{\text{ch}(\delta_k \cdot x)}{\text{ch}(\delta_k \cdot \frac{w}{2})} + \overline{f_{zk}^l} \cdot \frac{\text{sh}(\delta_k \cdot x)}{\text{sh}(\delta_k \cdot \frac{w}{2})} \right] \cdot \sin c\left(\lambda_k \cdot \frac{d}{2}\right) \cdot \cos(\lambda_k \cdot z) \right\}, \quad (30a)$$

$$\overline{e_{zk}^l} = \left[\frac{R_s^l + L_s^l}{M_s^l} - 2\left(\frac{\lambda_k}{\delta_k}\right)^2 \right], \quad (30b)$$

$$\overline{f_{zk}^l} = \frac{R_s^l - L_s^l}{M_s^l}, \quad (30c)$$

with $\lambda_k = k\pi/d$.

It should be noted that if $M_s^l = L_s^l = R_s^l$ then (30) is identical to the relation provided in [7]. Moreover, when $\alpha = 0$ (viz., $\sigma_{mp} = 0$ S/m and/or $f \cong 0^+$ Hz) the field distribution is equivalent to the excitation magnetic field.

2.4.5. Resultant Eddy-Current Density

From the general assumptions, and using the complex notation, the components of resultant eddy-current $\mathbf{J} = \{J_x; 0; J_z\}$ in massive conductive parts can be written as

$$J_x = \Re\{\overline{J_x} \cdot e^{j\omega \cdot t}\}. \quad (31a)$$

$$J_z = \Re\{\overline{J_z} \cdot e^{j\omega \cdot t}\}. \quad (31b)$$

Using $\mathbf{J} = \nabla \times \mathbf{H}$, the complex components of $\mathbf{J}$ in Cartesian coordinates (x, z) can be deduced by

$$\overline{J_x} = \frac{\partial \overline{H_{\sigma y}}}{\partial z}, \quad (32a)$$

$$\overline{J_z} = \frac{\partial \overline{H_{\sigma y}}}{\partial x}, \quad (32b)$$

which leads to

$$\overline{J_x} = H_s^l \cdot \left\{ -\alpha \cdot \frac{\text{sh}(\alpha \cdot z)}{\text{ch}(\alpha \cdot \frac{d}{2})} + \sum_{k=1,3,\dots}^{\infty} \lambda_k \cdot \left[\overline{e_{zk}^l} \cdot \frac{\text{ch}(\delta_k \cdot x)}{\text{ch}(\delta_k \cdot \frac{w}{2})} + \overline{f_{zk}^l} \cdot \frac{\text{sh}(\delta_k \cdot x)}{\text{sh}(\delta_k \cdot \frac{w}{2})} \right] \cdot \sin c\left(\lambda_k \cdot \frac{d}{2}\right) \cdot \sin(\lambda_k \cdot z) \right\} \quad (33a)$$

$$\overline{J_z} = H_s^l \cdot \sum_{k=1,3,\dots}^{\infty} \delta_k \cdot \left[\overline{e_{zk}^l} \cdot \frac{\text{sh}(\delta_k \cdot x)}{\text{ch}(\delta_k \cdot \frac{w}{2})} + \overline{f_{zk}^l} \cdot \frac{\text{ch}(\delta_k \cdot x)}{\text{sh}(\delta_k \cdot \frac{w}{2})} \right] \cdot \sin c\left(\lambda_k \cdot \frac{d}{2}\right) \cdot \cos(\lambda_k \cdot z) \quad (33b)$$

2.4.6. Comparing with 3-D FEA

The validation of the 2-D analytical model based on the Maxwell–Fourier method has been realized using Cedrat’s Flux3D software package by using the application “Harmonic State 3-D” [32].

The analytical solution of $H_{\sigma y}$ and $J = \{J_x; 0; J_z\}$ have been computed with a finite number of spatial harmonic term $2 \cdot K_{\max} - 1 = 241$. The 3-D FEA mesh consists of 32,169 surface and 93,031 volume elements of second order. Figure 13 shows the consumption time for the hybrid model versus k for only one operating point. By using the high discretization (i.e., $k = 24$) in the 2-D generic MEC for the BCs of the 2-D analytical model, the computation time for the hybrid model is greatly reduced as short as 6.5 s, whereas the 3-D FEA requires as much as 30 s. The proposed approach can thus reduce the computation time by approximately 4.6-fold compared to 3-D FEA.

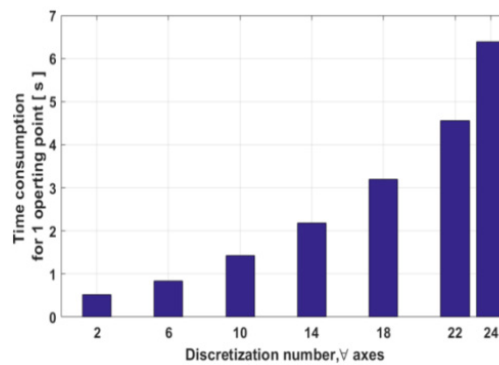


Figure 13. Consumption time for the hybrid model according to k (viz., the discretization number in massive conductive parts) for only one operating point.

Figure 14 shows the magnetic field distribution considering the skin effect on a 2-D grid parallel to the xz -plane located in the middle of the massive conductive part in the y -axis. The waveforms of $H_{\sigma y}$ have been calculated with $I_{\max} = 7.78 \text{ A}$ at $t = 0 \text{ s}$ and $h_{mp} = 6 \text{ mm}$ for two values of electrical frequency (viz., 50 Hz and 1600 Hz). Also for the same conditions, the evolution of the resultant eddy-current density is given in Figure 15. There is a very good agreement of the results given by analytic and numeric calculation. The electrical frequency effect on the behavior of $H_{\sigma y}$ and J can be clearly seen. In these figures, it can be seen that the skin effect appears slightly at 50 Hz, contrary to 1600 Hz where the massive conductive part act as a barrier to the crossing flux. The error order is less than 16% for $H_{\sigma y}$ (viz., 12% at 50 Hz and 16% at 1600 Hz) and less than 4% for J (viz., 1% at 50 Hz and 4% for 1600 Hz).

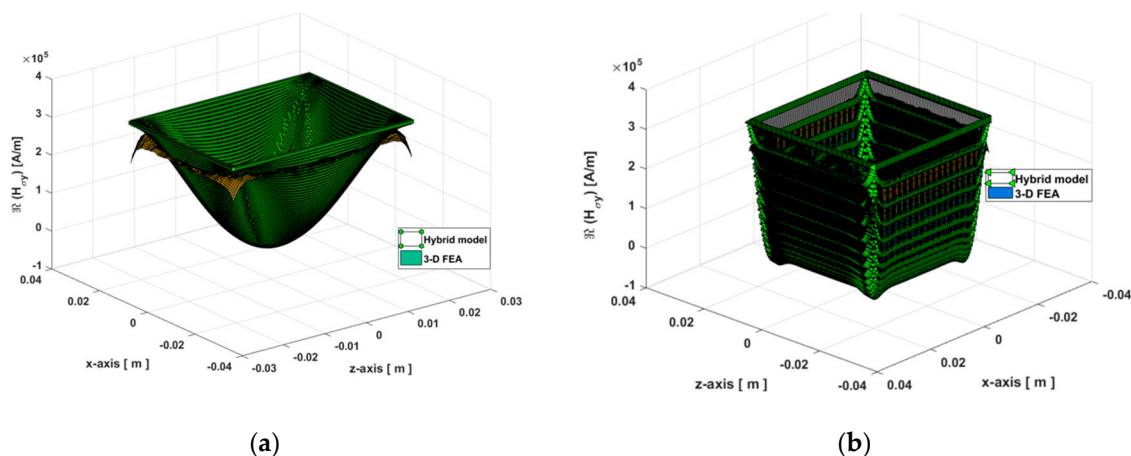


Figure 14. Evolution of $H_{\sigma y}$ in the massive conductive part with $I_{\max} = 7.78 \text{ A}$ at $t = 0 \text{ s}$ and $h_{mp} = 6 \text{ mm}$ for: (a) $f = 50 \text{ Hz}$, and (b) $f = 1600 \text{ Hz}$.

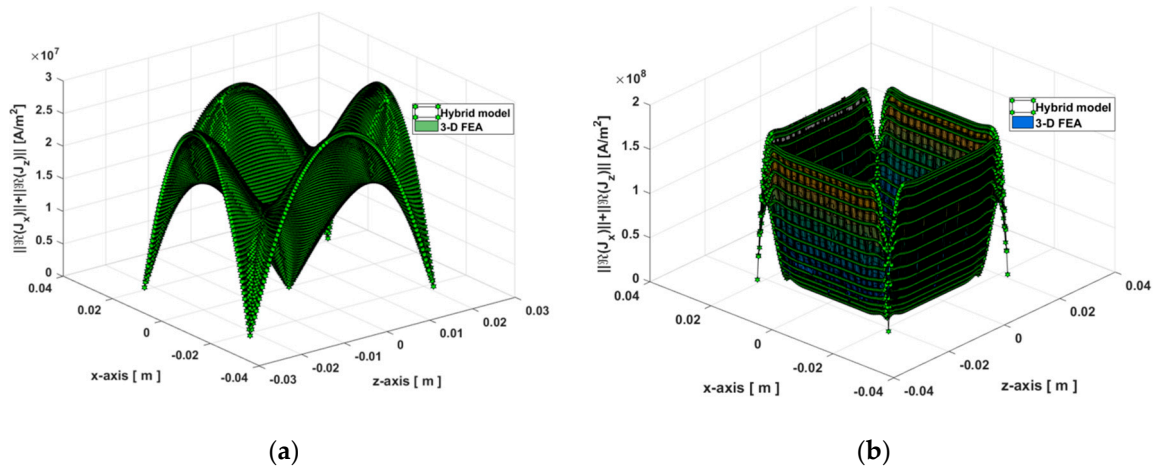


Figure 15. Evolution of J in the massive conductive part with $I_{\max} = 7.78 \text{ A}$ at $t = 0 \text{ s}$ and $h_{mp} = 6 \text{ mm}$ for: (a) $f = 50 \text{ Hz}$, and (b) $f = 1600 \text{ Hz}$.

3. 3-D Eddy-Current Loss Calculation

3.1. Mathematic Formulation

The instantaneous density of power flow Π at a point is defined by the Poynting vector [7]

$$\Pi = E \times H. \quad (34)$$

Using $J = \sigma \cdot E = \nabla \times H$, this power across a closed surface, in terms de complex vectors, is given by the instantaneous apparent power

$$\overline{s_{app}} = \oint_S \Pi \cdot dS = \frac{1}{\sigma_{mp}} \cdot \oint_S (J \times H) \cdot dS = p + j \cdot q \quad (35)$$

The real part of the surface integral of the complex Poynting vector gives the instantaneous ohmic losses, and the imaginary part the instantaneous magnetic energy.

From BCs at the edges of massive conductive parts (see Figure 10a), the average of $\overline{s_{app}}$ over an electrical cycle $T = 2\pi/\omega$ can be defined by, $\forall l$,

$$\overline{s_{app}} = \langle \overline{s_{app}} \rangle = \frac{h^l}{2\sigma_{mp}} \cdot \left\{ \int_{-w/2}^{w/2} 2 \cdot \overline{J_x} \cdot \overline{H_{oy}}^* \Big|_{z=-d/2} \cdot dx - \int_{-d/2}^{d/2} \left(\overline{J_z} \cdot \overline{H_{oy}}^* \Big|_{x=-w/2} - \overline{J_z} \cdot \overline{H_{oy}}^* \Big|_{x=w/2} \right) \cdot dz \right\} \quad (36)$$

where $h^l = h_{mp} / (2 \cdot Nd_3^y)$ is the layers thickness in the y-axis of the massive conductive part.

After the development, by substituting (30) and (33a) into (36), the average density of power flow is then given by, $\forall l$,

$$\overline{s_{app}} = P^l + j \cdot Q^l = \frac{h^l \cdot (H_s^l)^2}{\sigma_{mp}} \cdot \left\{ w \cdot \alpha \cdot \frac{\text{sh}(\alpha \cdot \frac{d}{2})}{\text{ch}(\alpha \cdot \frac{d}{2})} + \sum_{k=1,3,\dots} \frac{2\delta_k}{d \cdot (\lambda_k)^2} \cdot \left[\frac{(e_{zk}^l)^2 \cdot \frac{\text{sh}(\delta_k \cdot \frac{w}{2})}{\text{ch}(\delta_k \cdot \frac{w}{2})}}{\dots + (f_{zk}^l)^2 \cdot \frac{\text{ch}(\delta_k \cdot \frac{w}{2})}{\text{sh}(\delta_k \cdot \frac{w}{2})}} \right] \right\} \quad (37)$$

It should be noted that $P = \sum_i P^l$ only represents the 3-D eddy-current losses in massive conductive parts.

3.2. Experimental and Numerical Validations

In what follow, the analytical results are obtained by applying only one BC (viz., $M_s^l = L_s^l = R_s^l$) and also by applying three different BCs. The discretization impact of 2-D generic MEC in massive conductive parts is also discussed. The 3-D eddy-current loss results given by the model coupling are compared with those obtained by 3-D FEA and experimental tests. The experimental and numerical validations were performed at $f = 50$ Hz for two thicknesses in aluminum, viz., 6 mm and 10 mm.

3.2.1. Experimental Acquisition [33]

The eddy-current losses are calculated by using the separation of losses method. Firstly, the active power of the U-cored static electromagnetic device without the massive conductive parts is measured, then the active power after the insertion of massive conductive parts is measured. The difference between these two active powers gives the eddy-current losses created by the sinusoidal variation of the magnetic field in massive conductive parts.

3.2.2. Validation of Model Coupling with $M_s^l = L_s^l = R_s^l$

Figure 16 represent the evolution of P according to I_{max} when only the medium BC is applied over the edges of the massive conductive part (i.e., $M_s^l = L_s^l = R_s^l$) for $h_{mp} = 6$ mm and $h_{mp} = 10$ mm. The analytical results give a good agreement with 3-D FEA and experimental results.

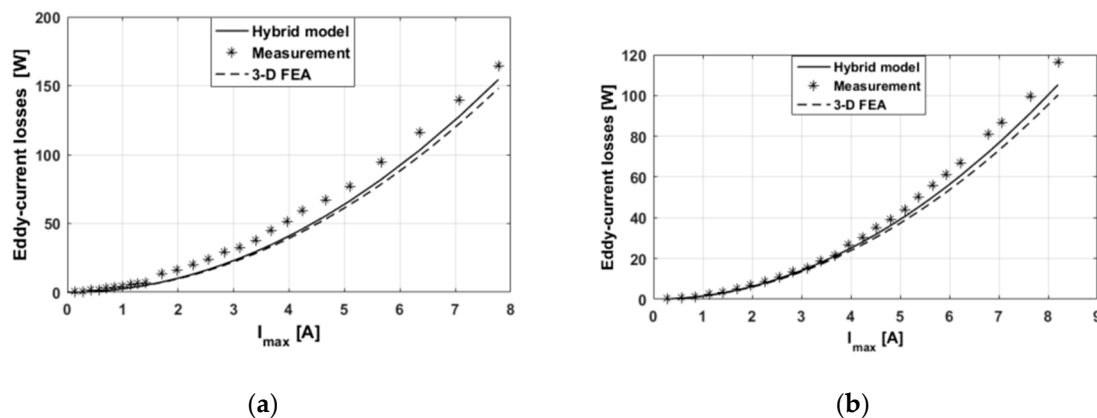


Figure 16. Validation of eddy-current losses (analytical, numerical, and experimental) in massive conductive parts versus I_{max} with $M_s^l = L_s^l = R_s^l$ for: (a) $h_{mp} = 6$ mm and (b) $h_{mp} = 10$ mm.

The difference between the analytical and experimental results can be linked to, $\forall h_{mp}$: (i) the experimental method, (ii) the electrical conductivity variation due to the temperature rise, and (iii) non-homogeneous BCs in the 2-D analytical model. For the experimental method, the use of analogue measurement instruments can affect the values obtained. In the 2-D analytical model developed, the electrical conductivity is assumed constant and invariant according to the temperature. In reality, the temperature variation influences the electrical conductivity values, and therefore the eddy-current losses in the massive conductive part due to the eddy-current reaction field. Then, it is interesting to note that the development of a magneto-thermal model would improve the error between the analytical and experimental results. Non-homogeneous BCs at the edges of the massive conductive part related to magnetic leakages (see Figure 10) affect to the distribution of the magnetic field H_{oy} inside the massive conductive part, and therefore the eddy-current losses. However, in [38], a 3-D generic MEC considering the skin effect would improve the volumic eddy-current loss calculation and observe the magnetic reaction field influence of the massive conductive parts on the magnetic circuit of the U-cored static electromagnetic device.

3.2.3. Validation of Model Coupling with $M_s^l \neq L_s^l \neq R_s^l$

To study the BCs influence, three different BCs at the edges of massive conductive parts are applied (i.e., $M_s^l \neq L_s^l \neq R_s^l$) (see Figure 11), and the results comparison are given in Figure 17. The results show a good agreement with experimental results compared to those obtained with $M_s^l = L_s^l = R_s^l$. The computation time is still acceptable even in high discretization.

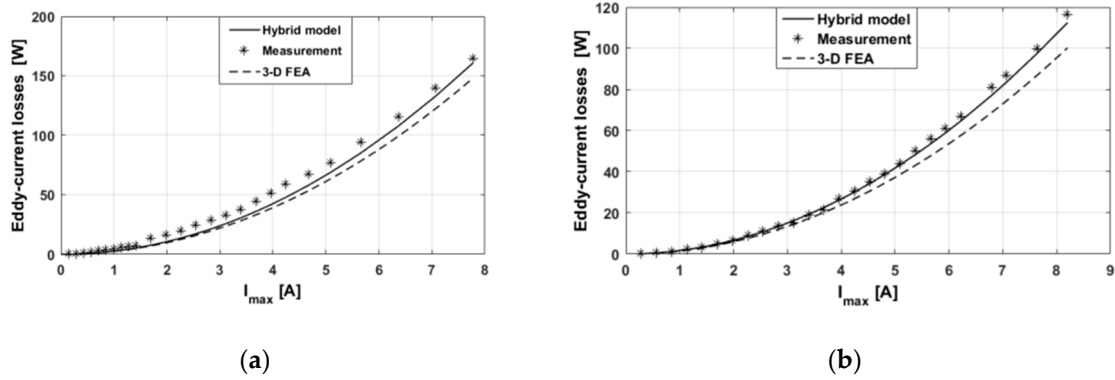


Figure 17. Validation of eddy-current losses (analytical, numerical, and experimental) in massive conductive parts versus I_{max} with $M_s^l \neq L_s^l \neq R_s^l$ for: (a) $h_{mp} = 6$ mm and (b) $h_{mp} = 10$ mm.

Figure 18 shows the normalized root-mean-square deviation (NRMSD) according to the discretization number in both axes (i.e., x- and y-axis). The NRMSD formulation, for all studied currents (see Table 2), is defined by

$$NRMSD = \frac{RMSD}{p_{N_1}^{meas} - p_1^{meas}} \text{ with } RMSD = \sqrt{\frac{\sum_{i=1}^N (p_i^{meas} - p_i^{anal})^2}{N}} \quad (38)$$

where N is the total number of currents used in experimental measures, p_i^{meas} and p_i^{anal} are respectively the eddy-current losses obtained analytically and experimentally for the i th current. It can be remarked that NRMSD decreases by increasing the discretization number, $\forall h_{mp}$. With the discretization used in (21), NRMSD is equal to 4.5% for 6 mm and 3% for 10 mm.

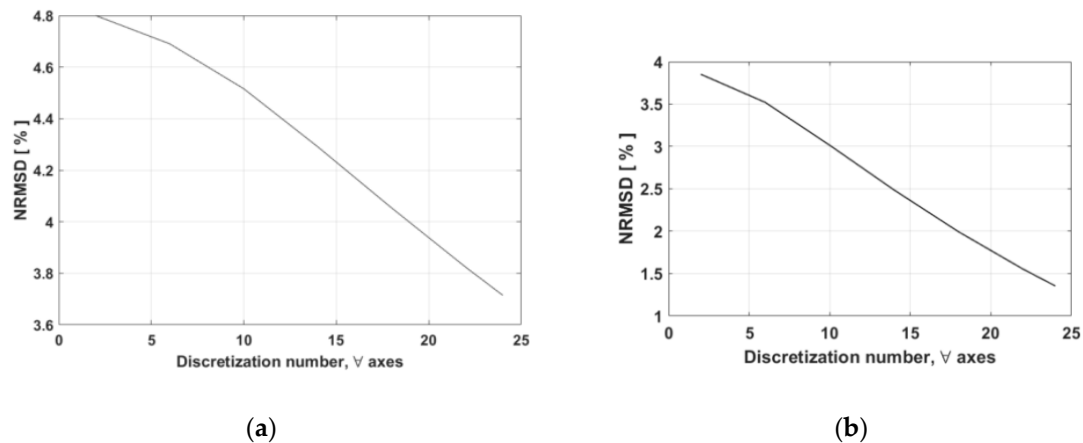


Figure 18. NRMSD according to the discretization number in both axes (i.e., x- and y-axis) for: (a) $h_{mp} = 6$ mm and (b) $h_{mp} = 10$ mm.

The eddy-current losses were calculated on different layers in the y-axis of massive conductive parts. Figure 19 shows the evolution of P^l at the top, middle, and bottom of the massive conductive

part for $h_{mp} = 6$ mm with a discretization Nd_3^y . It is interesting to note that the eddy-current losses present a non-uniform distribution depending on the height of the massive conductive part. This can lead to a non-uniform temperature distribution on the massive part. According to the magnetic field distribution in the y -axis (see Figure 10), the level of eddy-current losses is higher at the bottom of the massive conductive part. This is due to the magnetic flux density which presents a high level compared to the medium layer.

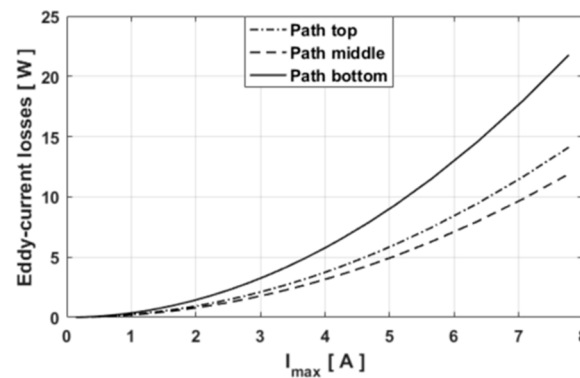


Figure 19. Evolution of the eddy-current losses in the massive conductive part for $h_{mp} = 6$ mm according to the layer in the y -axis.

4. Conclusions

In this work, a 3-D eddy-current losses model is developed, combining a 2-D generic MEC with a 2-D analytical model based on the Maxwell–Fourier method (i.e., the formal resolution of Maxwell’s equations by using the separation of variables method and the Fourier’s series). This model coupling has been applied to a U-cored static electromagnetic device [33]. The 2-D generic MEC determines the magnetic flux density distribution in massive conductive parts without the skin effect (i.e., without the eddy-current reaction field). The 2-D analytical model based on the Maxwell–Fourier method calculates the magnetic field distribution in massive conductive parts considering the skin effect as well as the resultant eddy-current density. BCs imposed on the 2-D analytical model are equivalent to the magnetic field of the MEC. The magnetic field distribution for various models is validated with 2-D and 3-D FEA. Experimental tests and 3-D FEA have been compared with the proposed approach for massive conductive parts in aluminum. For an operating point, the computation time is divided by 4.6 with respect to 3-D FEA. The study of the homogenous and non-homogenous BCs on the edges of massive conductive parts has been analyzed. Moreover, according to the 2-D MEC discretization, the model coupling is able to give more or less accurately the behavior of the magnetic field and eddy-current distribution in the massive conductive part.

The magnetic flux density repartition in different paths parallel to the x -axis present different variations. Consequently, the eddy-current losses present a non-uniform distribution over the massive conductive part. This can lead to predicting the temperature distribution over the conducting region while designing thermal components of the electromagnetic devices.

Furthermore, one advantage of this coupling model would be its exploitation in studies of PMSMs with(out) circumferential and/or axial PMs segmentation in order to reduce the computation time, which remains a major problem in this numerical method.

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