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Theoretical Estimates of the Critical Reynolds Number in the Flow around the Sphere on the Basis of Theory of Stochastic Equations and Equivalence of Measures

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Abstract: The aim of this investigation is to show the solution for the critical Reynolds number in the flow around the sphere on the basis of theory of stochastic equations and equivalence of measures between turbulent and laminar motions. Solutions obtained by numerical methods (DNS, LES, RANS) require verification and in this case the theoretical results have special value. For today in the scientific literature, there is J. Talor's implicit formula connecting the critical Reynolds number with the parameters of the initial fluctuations in the flow around the sphere. Here the derivation of the explicit formula is presented. The results show a satisfactory correspondence of the obtained theoretical dependence for the critical Reynolds number to the experiments in the flow around the sphere.

Keywords: equivalence of measures; stochastic equations; critical Reynolds number; critical point; the flow around the sphere



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1. Introduction

The main ideas of the theory of onset of turbulence are described in [1–11].

The theory of Landau describes the onset of turbulence as quasi-periodic motion which is realized by multiple frequency doubling [1]. The theory of strange attractors and the corresponding differential equations are presented in the original papers of Lorenz, E.N., Feigenbaum, M., Ruelle, D., Takens, F. [3–5]. The main points of statistic theory of turbulence are presented in the articles of Kolmogorov A.N. [6–9].

The idea about criteria of the relative degree of the ordering of states in self-organization processes is presented in papers of Struminskii, V.V. and Klimontovich, Y.L. [10,11].

Further complex mathematical and numerical methods for solving of the Navier–Stokes equation using the theory of strange attractors and the theory of solitons are shown in [12–24]. The theory of solitons on the basis of the Korteweg–de Vries equations and the theory of chaos are presented in [12–15]. The main idea about equations of Kolmogorov corresponding to a two-dimensional stochastic Navier–Stokes system is presented in [16]. The task of determination of the correlation dimension strange attractors was studied in [17–20]. The fractals in the turbulent flows and Kolmogorov entropy were studied in [21–24].

Pure numerical methods such as DNS for solution of Navier–Stokes equations are presented in [25–28]. The statistical and stochastic equations for research of the turbulence are presented in [29–35].

It was also hypothesized that the flow region with a large absolute value of the Jacobian $\partial(u,v)/\partial(x,y)$ generates intense pressure waves. These waves probably give rise to turbulent pulsations [34–36].

In essence, the theoretical solutions for the critical Reynolds number were done by using well-known ratio based on of the theory of dimension and experimental data [36–38].

However, experimental formulas require the initial conditions for concrete flows. Thus, a new theory for the determination of analytical formulas for the critical number of onset of turbulence is needed.

The latest results of the theory of stochastic equations and the theory of equivalence of measures between deterministic and random states have allowed, for the first time, the derivation of analytical dependences for friction coefficients in a laminar–turbulent transition in cases of the isothermal flows on the smooth flat plate and in the round tube [39,40].

It is obvious that the experiment or numerical study of this phenomenon should be based on conservation laws, and the methods of conducting experiments and solving equations of these laws should have verification tools. Such a tool can be considered as being the third research methodology: theoretical. Thus, such methodology is primarily aimed at creating a new physical and mathematical toolkit, which allows the obtaining of analytical results. This is especially important for those phenomena for which the energy–space–time properties were identified: the uncertainty relation. As is known, this ratio was initially determined for quantum mechanics, which is based on wave and quantum ideas. Recent studies have shown the correctness of the ratio of uncertainties in the stochastic mechanics of liquid and gas. Thus, the generation of energy in random processes in both quantum mechanics and classical mechanics determines the existence of the uncertainty relation. Therefore, this ratio determines a criterion or feature for verifying natural and numerical experiments. The importance and materiality of such the tool in both of these methodologies is obvious and noted repeatedly. Therefore, the creation of a theoretical tool, which makes it possible to qualitatively and quantitatively evaluate the most important parameters for practice while taking into account the uncertainty relation, determining, among other things, the possibility of verifying research in natural and numerical experiments. The development of such a theoretical apparatus for the study of a natural phenomenon requires repeated checks of this apparatus by calculating the characteristics of various types of the phenomenon under study.

These new results have been made possible as a result of successive advances in theoretical physics related to turbulence. Namely the theoretical dependencies for first and second critical Reynolds number and dependences for profiles of averaged velocity and temperature fields in the boundary layers [41–47] were obtained. Moreover, the friction and heat-transfer coefficients and second-order correlations were derived [48–52]. Besides the formulas for Reynolds analogy and formulas of the correlation dimension of an attractor in the boundary layer on the flat plate and in the tube were received [53–58].

Note that the uncertainty relation in the process of turbulence generation was received [59,60]. For this case the spectral function $E(k)_j$ depends on wave numbers k for interval of generation of turbulence in the form of $E(k)_j \sim k^n$, $n = -1.2 \div -1.5$.

Furthermore, theoretical solutions for the spectral function $E(k)_j$ in cases of Kolmogorov and dissipation intervals [59–63] were derived. [59–63].

Other classes of fluid motions were also studied. The turbulent flow in the plane jet, the flow near a rotating disk and the motion between rotating coaxial cylinders are presented in [64–66].

Here, the analytical solutions for the critical Reynolds number and for the critical point in the case of the motion around the sphere are presented.

2. Equations of Conservation for the Isothermal Stochastic Process

The equations were derived in [39–47] and for isothermal movement without the external forces take the following form:

The equation of mass (continuity):

$$\frac{d(\rho)_{col_{st}}}{d\tau} = -\frac{(\rho)_{st}}{\tau_{cor}} - \frac{d(\rho)_{st}}{d\tau}, \quad (1)$$

The momentum equation:

$$\frac{d(\rho u_i)_{colst}}{d\tau} = \text{div}(\tau_{i,j})_{colst} + \text{div}(\tau_{i,j})_{st} - \frac{(\rho \vec{U})_{st}}{\tau_{cor}} - \frac{d(\rho \vec{U})_{st}}{d\tau} \quad (2)$$

And the energy equation:

$$\frac{dE_{colst}}{d\tau} = \text{div}(u_i \tau_{i,j})_{colst} + \text{div}(u_i \tau_{i,j})_{st} - \left(\frac{E_{st}}{\tau_{cor}} \right) - \left(\frac{dE_{st}}{d\tau} \right) \quad (3)$$

Here, $u_i, u_j, u_l, E, \rho, \mu, \tau, \tau_{i,j}$ are the velocity components in the directions x_i, x_j, x_l ($i, j, l = 1, 2, 3$); the energy, the density, the dynamic viscosity, the time and the stress tensor $\tau_{i,j} = P + \sigma_{i,j}$, $\sigma_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \delta_{ij} \left(\zeta - \frac{2}{3} \mu \right) \frac{\partial u_l}{\partial x_l}$ here i, j are the tensorial notations, $\delta_{ij} = 1$ if $i = j$, $\delta_{ij} = 0$ for $i \neq j$. P is the pressure of liquid or gas. The subscript “st” refers to the components, which are actually stochastic. The subscript “col st” refers to the components, which are actually deterministic.

3. Stochastic Equations for Critical Reynolds Number

The flow around a sphere, along with flows in the boundary layer on a plate, in a pipe and in a jet, belongs to the classical ones. Note that the question of the transition from a laminar flow to a turbulent flow around a sphere was considered in a number of works [37,67–69]. The example under consideration, in contrast to the previously considered flows, is characterized by the fact that before the process of transition of a laminar, deterministic flow to a turbulent one occurs, the separation of the boundary layer from the surface of the sphere occurs. As is known, in this case the separation process is not accompanied by a simultaneous transition to turbulence. Thus, on the basis of the already defined equivalence relation of measures, it is necessary to determine the critical Reynolds number in the boundary layer when flowing around a sphere.

To decide Equations (1)–(3) the correlator $D_{N,M}$ was derived in [39–46] as the definition of equivalency of measures between laminar movement and turbulent movement. The results of using of the correlator $D_{N,M}$ applying to (1)–(3) are the sets of stochastic equations for four space–time areas: (1) the onset of generation (subscript 1,0 or 1); (2) the generation (subscript 1,1); (3) the diffusion (1,1,1) and (4) the dissipation of the turbulent fields.

The correlator $D_{N,M}$ for all four space–time regions in the critical point of space–time $r_i \rightarrow r_c; \Delta\tau_i \rightarrow \tau_c$ for the parameter $m_i \rightarrow m_c$ can be written in form

$$\sum_j D_{M,N}(r_c; m_{cj}; \tau_c) = \sum_j \sum_i \lim_{m_i \rightarrow m_{cj}} \lim_{r_i \rightarrow r_c} \lim_{\Delta\tau_i \rightarrow \tau_c} \{ m(T^M Z^* \cap T^N Y^*) - R_{T^M Z^* T^N Y^*} \cdot m(T^M Z^*) \} = 0.0. \quad (4)$$

The binary intersections are considered between subsets Y, Z, W , in space $X = Y + Z + W$. The subsets of Y, Z, W have the name “extended to X ”, if the measures $m(Y), m(Z), m(W)$ have the property [32–38]:

$$\begin{aligned} m(Y) = m(Y^*) &= m(T^n Y) + \bigcup_{k=0}^{k=n-1} m(T^k(G_1^{n-k})) \text{ and wandering subset } \bigcup_{k=0}^{k=n-1} m(T^k(G_1^{n-k})) \subset Y; \\ m(Z) = m(Z^*) &= m(T^n Z) + \bigcup_{k=0}^{k=n-1} m(T^k(G_2^{n-k})) \text{ and wandering subset } \bigcup_{k=0}^{k=n-1} m(T^k(G_2^{n-k})) \subset Z; \\ m(W) = m(W^*) &= m(T^n W) + \bigcup_{k=0}^{k=n-1} m(T^k(G_3^{n-k})) \text{ and wandering subset } \bigcup_{k=0}^{k=n-1} m(T^k(G_3^{n-k})) \subset W. \end{aligned}$$

Index j is determined parameters m_{cj} ($j = 3$ means: mass, momentum, energy). This correlation function produces the system of equations of equivalent measures $|m(T^M Z)| = (R_{T^M Z T^N Y})_{(n,m)} |m(T^N Y)|, 0 < (R_{T^M Z T^N Y})_{(n,m)} \leq 1$.

Here, $(R_{TMZTY})_{(n,m)}$ is a fractal correlation function, and then we assume it is equal to the unit to obtain analytical solutions. Therefore, for the pair $(N,M) = (1,0)$ we have $|m(Z)| = (R_{ZTY})_{(n,m)}|m(TY)|$, and for $(N,M) = (1,1)$, $|m(TZ)| = (R_{TZTY})_{(n,m)}|m(TY)|$. Here, T^n is a conservative transformation of X for all n , then there exists $n > n_{cr}$, such that there T^n is dissipation and transformation for $Y \subset X$ and $Z \subset X$. Then to the set X corresponds the value of the total energy of the stochastic field $\cup_i^{i=n}(E_i)_{gst}$; to a subset $Y \subset X$ corresponds the value of energy of deterministic component of the stochastic field $\cup_i^{i=n}(E_i)_{colst}$; to the wandering subset G_1^n extended in subset $Y \subset X$, corresponds $(\delta Q + \delta L)_{(col)int}$; to the subset $Z \subset X$ of the measure $m(Z) > 0$ corresponds the value of the stochastic component of energy $\cup_i^{i=n}(E_i)_{st}$; to the wandering subset G_2^n extended in subset $Z \subset X$ corresponds $(\delta Q + \delta L)_{(st)int}$; to the subset $W \subset X$ and to the wandering subset G_3^n corresponds to the the value of the $\cup_i^{i=n}(E_i)_{corst}$. Furthermore, to the transformation T^n of the set or subset corresponds the differential operators $\left\{d; \frac{d}{d\tau}\right\}$. Here $(\delta Q + \delta L)_{(st)int}$ is the sum of elemental heat and elementary work.

The value of $R_{1TMZ*TY*}$ for each of four space–time areas is equal to 1 here. The subscript j denotes the parameters m_{cj} ($j = 3$ means mass, momentum and energy). The correlator $D_{M,N}(r_c; m_{cj}; \tau_c) = D_{1,1}(r_c; m_{cj}; \tau_c)$ for the pair $(N,M) = (1,1)$ gives the following equations: $(d\Phi_{colst})_{1,1} = -R_{1,1}(d\Phi_{st})$, $\left(\frac{d(\Phi_{colst})}{d\tau}\right)_{1,1} = -R_{1,1}\left(\frac{d\Phi_{st}}{d\tau}\right)$. Here, Φ is the substantial quantity (mass (density ρ), momentum ($\rho\mathbf{U}$), energy (E)). The fractal coefficients $R_{1,0TMZ*TY*} = R_{1,0}$, $R_{1,1TMZ*TY*} = R_{1,1}$ are taken to be equal in unity. The subscripts “cr” or “c” refer to the critical point r (x_{cr} , τ_{cr}) or r_c , which is the space–time point of the onset of the interaction between the deterministic motion and the random motion, which leads the turbulence. Therefore, according to the above, the system of equations corresponding to the beginning of the transition (the space–time area 1), which determines the critical Reynolds number for the case of an incompressible flow, has the next form:

$$\frac{d(\rho)_{colst}}{d\tau} = -\frac{(\rho)_{st}}{\tau_{cor}} \quad (5)$$

$$\begin{cases} \frac{dE_{colst}}{d\tau} = -\left(\frac{E_{st}}{\tau_{cor}}\right) \\ div(u_i \tau_{i,j})_{colst} = -\left(\frac{E_{st}}{\tau_{cor}}\right) \end{cases}$$

The solution to the problem of laminar, deterministic, motion on the surface of a sphere is presented, in particular, in I.G. Schlichting [37]. Following the provisions of the stochastic theory of turbulence, the critical Reynolds number is determined by the relation (6), within the left side of which it is necessary to put a solution for laminar flow over the surface of a sphere.

$$div(u_i \tau_{i,j})_{colst1} = \left|\frac{E_{st}}{\tau_{cor}^0}\right|_{1,0} \quad (6)$$

As is known, the velocity profile of a laminar boundary layer on a sphere is similar in representation to the Blasius profile, with the difference being that the profiles are presented as a function of the angle and coordinate. The profile of the laminar boundary layer before the separation can be written as

$$\frac{u_1}{U_0} = f\left[x_2\left(\frac{U_0}{\nu R}\right)^{0.5}\right]. \quad (7)$$

Here orthogonal coordinates x_1 and x_2 are chosen, the x_2 coordinate is directed along the normal to the surface of the sphere along the radius, R . Now, taking into account the approximation in the region of constant velocity gradient, it is possible to write

$$\frac{u_1}{U_0} = \left[K_\phi x_2 \left(\frac{U_0}{\nu R} \right)^{0.5} \right]. \quad (8)$$

Then, let us write the speed differential; here, K_ϕ is a constant

$$du_1 = d \left(U_0 \left[K_\phi x_2 \left(\frac{U_0}{\nu R} \right)^{0.5} \right] \right) \quad (9)$$

or

$$du_1 = d \left(U_0 \left[K_\phi x_2 \left(\frac{U_0}{\nu R} \right)^{0.5} \right] \right) = K_\phi U_0 \left(\frac{U_0}{\nu R} \right)^{0.5} dx_2. \quad (10)$$

Then, given that the angle is fixed, we write that

$$\frac{du_1}{dx_2} = d \left(U_0 \left[K_\phi x_2 \left(\frac{U_0}{\nu R} \right)^{0.5} \right] \right) = K_\phi U_0 \left(\frac{U_0}{\nu R} \right)^{0.5}. \quad (11)$$

Obviously, for a deterministic (laminar) flow $\frac{du_1}{dx_2} > \frac{du_1}{dx_1}$

$$\text{div}(u_i \tau_{i,j})_{col_{st1}} = \mu 2 K_\phi^2 U_0^2 \left(\frac{U_0}{\nu D} \right). \quad (12)$$

Then, the equivalence of measures between deterministic and stochastic moves is written as follows

$$\left(2 \mu K_\phi^2 U_0^2 \left(\frac{U_0}{\nu D} \right) \right) = \frac{E_{st}}{\tau_{cor}^0}. \quad (13)$$

From where it is possible to determine the dependence for the dimensionless number at which the equivalence of measures occurs, named in hydrodynamics as the critical Reynolds number for the flow in the boundary layer for the corresponding values of the time correlation: $(\tau_{cor}^0)_i, (\tau_{cor}^0)_2, (\tau_{cor}^0)_3, \tau[\text{div}(u_i)]^{-1}_{det}$.

For $(\tau_{cor}^0)_1 = L/(E_{st}/\rho)$ the Reynolds number will be written as

$$(Re_D)_1 = 2 \cdot [K_\phi^2] \left(\frac{U_0}{(E_{st}/\rho)^{0.5}} \right)^4 Re_{st}. \quad (14)$$

For the time correlation $(\tau_{cor}^0)_2 = L^2/\nu$ the Reynolds number is

$$(Re_D)_2 = 2 \cdot [K_\phi^2] \left(\frac{U_0}{(E_{st}/\rho)^{0.5}} \right)^4 Re_{st}^2. \quad (15)$$

Accordingly, for the value $(\tau_{cor}^0)_3 = \nu/(E_{st}/\rho)$

$$(Re_D)_3 = 2 \cdot [K_\phi^2] \left(\frac{U_0}{(E_{st}/\rho)^{0.5}} \right)^4, \quad (16)$$

L is the turbulence scale taken along the x_2 . From these expressions, as well as for the flow in a pipe, it is clear that the critical Reynolds number $(Re_D)_i$ is a local parameter determined in the region of space $r(x_1, x_2)_{cr}$. The perturbation parameters in (14)–(16) are

$\frac{U_0}{(E_{st}/\rho)^{0.5}}, Re_{st} = \frac{L\sqrt{E_{st}/\rho}}{\nu}$. The coefficient $[K_\phi]$ is determined by the tangent of the slope angle at the critical point $(x_2)_{cr}$.

4. The Definition of the Critical Point

The definition of the critical point $(x_2)_{cr}$, as well as for the flow in the pipe, will be written using the ergodic theory:

$$\int_{-\Delta V/2}^{+\Delta V/2} d(E_{colst})_{1,0} = \int_X dE_{st}, \quad (17)$$

$$E_{st} = E_{st}(\vec{x}_i, \tau_i, m_i) < \infty. \quad (18)$$

E_{st} is the stochastic energy component in the space X , measure $m(E_{st}) < \infty$. Then, it is possible find next expressions:

$$\int_X dE_{st} = \frac{1}{L} \int_L E_{st} \delta((x_2) - x_2) dL = \frac{1}{\tau_{cor}^0} \int_\tau E_{st} \delta(\tau_{cor}^0 - \tau) d\tau = E_{st}, \quad (19)$$

$$\int_{-L/2}^{+L/2} d(E_{colst})_{1,0} = K_\phi^2 L \cdot (x_2)_{critic} \cdot (\rho U_0^2 / 2) \left(\frac{U_0}{\nu R} \right), \quad (20)$$

$$(x_2)_{critic}^\phi = 2 \left(\frac{E_{st}/\rho}{U_0^2} \right) \left(\frac{\nu}{U_0} \right) \frac{R}{L} \frac{1}{K_\phi^2}, \quad (21)$$

$$\left(\frac{x_2}{R} \right)_{critic}^\phi = 4 \left(\frac{E_{st}/\rho}{U_0^2} \right) \left(\frac{\nu}{U_0 D} \right) \frac{R}{L} \frac{1}{K_\phi^2}, \quad (22)$$

$$\left(\frac{x_2}{R} \right)_{critic}^\phi = 4 \cdot \left(\frac{E_{st}/\rho}{U_0^2} \right) \left(\frac{1}{Re_D} \right) \left(\frac{R}{L} \right) \frac{1}{K_\phi^2}. \quad (23)$$

Note that it may be of interest to make a similar estimate, when the laminar-motion velocity profile is determined by the motion of Taylor vortices but not by the profile for the boundary layer; however, this version of the definition is not used here.

5. Results of Estimates of the Critical Taylor Number

The implicit form of the Taylor formula for the first critical Reynolds number is presented as the following relationship [37,67–69]

$$Re_D = f \left[\left(\frac{(E_{st}/\rho)^{0.5}}{U_0} \right) \left(\frac{D}{L} \right)^{1/5} \right]. \quad (24)$$

Note that the conditions of the experiments [31,67–69] are next: the pulsation intensity $[(E_{st}/\rho)^{0.5}/U_0] = 6\text{--}0.5\%$, the relative magnitude of the turbulence scale (L/R) is $10^{-3}\text{--}10^{-4}$ and the experimental values of the critical Reynolds number $(Re)_{cr} \sim 80,000\text{--}300,000$.

Thus, the critical Reynolds number and critical point can be determined with using Formulas (14) and (23). The values of the pulsation intensity and the scale of turbulence may be determined from experiments: For $R = 0.15$ [m], $U = 5$ [m/c], the degree of turbulence intensity 4.5% $u' = 0.225$ [m/c], $L = 0.00011$ [m], $Re_{st} = 1.68$, $K_\phi = 0.3$. So, critical Reynolds number has a value

$$(Re_D)_{1critic} = 2 \cdot [0.3^2] (22.3)^4 \frac{0.225 \cdot 1.1 \cdot 10^{-4}}{1.47 \cdot 10^{-5}} = 2 \cdot 2.43 \cdot 10^5 \cdot 9.9 \cdot 10^{-2} \cdot 1.68 \approx 0.8 \cdot 10^5. \quad (25)$$

In accordance with experimental data [67–69], it is possible to estimate the values of a critical point in the boundary layer of the flow around the sphere:

$$\left(\frac{x_2}{R}\right)^{\phi}_{critic} = 4 \cdot \left(\frac{E_{st} \rho}{U_0^2}\right) \left(\frac{1}{Re_D}\right) \left(\frac{R}{L}\right) \frac{1}{K_{\phi}^2} =$$

$$4 \cdot 20.25 \cdot 10^{-4} \cdot 1 / (0.8 \cdot 10^5) \cdot 1.5 \cdot 10^{-1} / (1.1 \cdot 10^{-4}) \cdot 1 / 9 \cdot 10^{-2} =$$

$$81 \cdot 1.423 \cdot 10^{-9} \cdot (1.5/9) \cdot 10^5 \approx 1.54 \cdot 10^{-3}; \quad (26)$$

$$(x_2)^{\phi}_{critic} = 0.00154 \cdot 0.15 = 0.00024 \text{ [m]}. \quad (27)$$

It is possible to make an indirect comparison with experimental data for the critical point: the calculation result $L/(x_2)^{\phi}_{critic} = 1.1 \cdot 10^{-4} / 0.00024 \approx 0.46$ and at the same time the Von Karman constant is $k = 0.43\text{--}0.38$, so the difference is $\sim 10\text{--}15\%$.

Obviously, with a decrease in the degree to 0.5%, the scale of turbulence will also decrease. Then for 0.5%, degree of turbulence the critical Reynolds number has a value

$$(Re_D)_{1critic} = 2 \cdot [0.035^2] (200)^4 \frac{0.025 \cdot 4.5 \cdot 10^{-5}}{1.47 \cdot 10^{-5}} =$$

$$2 \cdot 16 \cdot 10^8 \cdot 12.25 \cdot 10^{-4} \cdot 6.8 \cdot 10^{-2} \approx 3.00 \cdot 10^5, \quad (28)$$

$$\left(\frac{x_2}{R}\right)^{\phi}_{critic} = 4 \cdot \left(\frac{E_{st} \rho}{U_0^2}\right) \left(\frac{1}{Re_D}\right) \left(\frac{R}{L}\right) \frac{1}{K_{\phi}^2} =$$

$$4 \cdot 25 \cdot 10^{-6} \cdot 1 / (3 \cdot 10^5) \cdot 1.5 \cdot 10^{-1} / (4.5 \cdot 10^{-5}) \cdot 1 / 1.225 \cdot 10^{-3} =$$

$$100 \cdot 0.33 \cdot 10^{-11} \cdot (1.5/5.51) \cdot 10^7 = 33 \cdot 10^{-4} \cdot 0.27 = 1.12 \cdot 10^{-3};$$

$$(x_2)^{\phi}_{critic} = 0.15 \cdot 1.12 \cdot 10^{-3} = 1.35 \cdot 10^{-4} \text{ [m]}. \quad (29)$$

The indirect comparison with experimental data for the critical point is in the next: the calculation result $L/(x_2)^{\phi}_{critic} = 4.5 \cdot 10^{-5} / 0.000135 = 0.33$ and at the same time the Von Karman constant is $k = 0.43\text{--}0.38$, so the difference is $\sim 15\text{--}20\%$.

It must be noted that in the original works of J. Taylor [37,67] there are no simultaneous comparisons of values for the calculated critical Reynolds number with experimental data and turbulent characteristics with any verified characteristic, such is the constant of Von Karman from the statistical theory of turbulence.

At the same time, in the present work, special attention is paid to this aspect, since such a multi-parameter comparison with experiments of both critical numbers and turbulent characteristics allows us not only to verify the proposed theoretical relationships, but also to confirm the general points of theory for determining turbulent characteristics on the basis of the stochastic equations and theory of equivalence of measures. Additionally, the results show that as the turbulence intensity decreases, the transition point approaches the solid surface of the sphere. Attempts to present the dependence of the critical Reynolds number on the scale and intensity of turbulence have been made repeatedly [35–37,70–86].

Results of the calculation in accordance with Formula (23) which is obtained on the basis of stochastic equations that are presented on the Figure 1 compared to experiments of Dryden, H. L., Kueth, A. M. and Dryden, H. L., Schubauer G. B. [68,69].

It should be noted that such a detailed experimental study of the dependence of the critical Reynolds numbers on turbulent fluctuations in the flow around a sphere, as far as the authors know, has not yet been carried out.

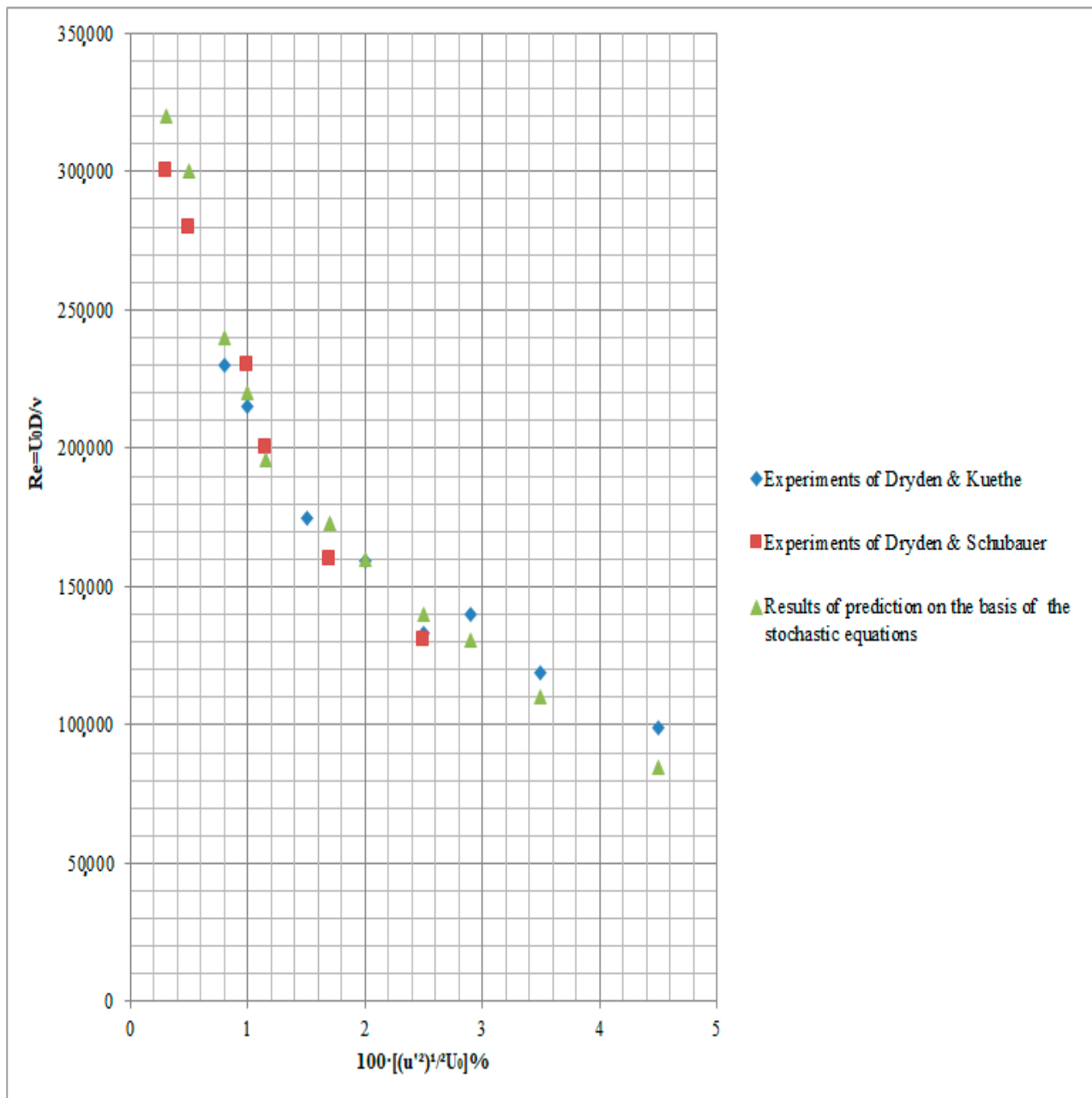


Figure 1. Comparison of calculation with experimental data for the critical Reynolds number on the degree of turbulence during the flow around the sphere [37,68,69].

6. Conclusions

In this article, the analytical Formula (14) for the critical Reynolds number and for the critical point (23) for the motion around the sphere are presented.

These results are obtained on the basis of the theory of stochastic equations for the continuum and the theory of the equivalence of measures between random and deterministic motions.

The calculation results using Equation (14), which are presented in Figure 1, show a satisfactory agreement with the experimental data. The analytical dependence of the critical Reynolds number on the degree of turbulence in a flow around a sphere show the possibility of theoretically predicting the transition to a turbulent regime in a boundary layer on a sphere in the range of the degree of turbulence 0.5–6%. In contrast to J. Taylor's

implicit formula connecting the critical Reynolds number with the parameters of the initial fluctuations in the flow around the sphere the new the explicit formula is presented (14).

On the basis of Formula (23), calculations of the values of the critical point in the boundary layer on a sphere show the same range of values as in the boundary layer on a smooth flat plate and in the boundary layer in a smooth round pipe $\left(\frac{x_2}{R}\right)^{\phi}_{critic} \approx 10^{-3}$.

The calculations show satisfactory agreement regarding the theoretical dependences for the critical Reynolds number with the experiments with an accuracy of up to 10%. The calculated values of constant von Karman using Equations (14) and (23) show the agreement up to 10–20% with the experimental von Kaman constant.

Therefore, the theoretical Dependences (14) and (23) may be used to check both experimental and calculation research of the transition to turbulence in the flow around the sphere. The importance of the obtained formulas lies in the numerous problems that occur during the movement of spherical bodies in the ocean and in the atmosphere, as well as in technical devices with flows of suspended solid and liquid spherical particles.

Moreover, as was mentioned in [39,40], such results which are presented in this article probably open up prospects both for the development of new experimental measuring instruments and for the development of a new calculation method—Direct Theoretical–Numerical Simulation (DTNS).

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