

Fuzzy Inference Full Implication Method Based on Single Valued Neutrosophic t-Representable t-Norm [†]

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Abstract: As a generalization of intuitionistic fuzzy sets, single-valued neutrosophic sets have certain advantages for solving indeterminate and inconsistent information. In this paper, we study the fuzzy inference full implication method based on a single-valued neutrosophic t-representable t-norm. Firstly, single-valued neutrosophic fuzzy inference triple I principles for fuzzy modus ponens and fuzzy modus tollens are shown. Then, single-valued neutrosophic $\mathcal{R}$ -type triple I solutions for fuzzy modus ponens and fuzzy modus tollens are given. Finally, the robustness of the full implication of the triple I method based on a left-continuous single-valued neutrosophic t-representable t-norm is investigated. As a special case in the main results, the sensitivities of full implication triple I solutions, based on three special single-valued neutrosophic t-representable t-norms, are given.

Keywords: single-valued neutrosophic set; single-valued neutrosophic t-representable t-norm; full implication triple I method



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1. Introduction

It is widely known that fuzzy reasoning plays an important role in fuzzy set theory. Particularly, the most basic forms of fuzzy reasoning are fuzzy modus ponens (FMP for short) and fuzzy modus tollens (FMT for short), which can be shown as follows [1,2]:

FMP (A, B, A^*): given fuzzy rule $A \rightarrow B$ and premise A^* , attempt to reason a suitable fuzzy consequent, B^* .

FMT (A, B, B^*): given fuzzy rule $A \rightarrow B$ and premise B^* , attempt to reason a suitable fuzzy consequent, A^* .

In the above models, $A, A^* \in F(Y)$ and $B, B^* \in F(Y)$, where $F(X)$ and $F(Y)$ denote fuzzy subsets of the universes X and Y , respectively.

The most famous method to solve the above models is the compositional rule of inference (CRI for short), which is presented by Zadeh [2,3]. However, the CRI method lacks clear logical semantics and reductivity. In order to overcome these shortcomings, Wang [1] proposed the fuzzy reasoning full implication triple I method, which can bring fuzzy reasoning into the framework of logical semantics [4]. In recent years, many scholars have studied the fuzzy reasoning full implication method. Wang et al. [5] provided unified forms for the fuzzy reasoning full implication method based on normal and regular implications. Pei [6] provided a unified form for the fuzzy reasoning full implication method based on the residual implication induced by left-continuous t-norms. Moreover, Pei [7] established a solid logical foundation for the fuzzy reasoning full implication method based on left-continuous t-norms. Liu et al. [8] provided a unified form for the solutions of the fuzzy reasoning full implication method. Luo and Yao [9] studied the fuzzy reasoning triple I method based on Schweizer–Sklar operators.

Although fuzzy set theory has been successfully applied in many fields, there are some defects in dealing with fuzzy and incomplete information. Turksen [10] proposed

interval-valued fuzzy sets, which represented a subinterval in $[0, 1]$ membership function. In recent years, some high-quality research results on interval-valued fuzzy reasoning have been achieved. Li et al. [11] extended CRI method on interval-valued fuzzy sets. Additionally, Luo et al. [12–15] studied the interval-valued fuzzy reasoning full implication triple I method and reverse triple I method based on the interval-valued-associated t-norm. Moreover, Luo et al. [16] studied the fuzzy reasoning triple I method based on the interval-valued t-representable t-norm.

Although the interval-valued fuzzy set has some advantages in dealing with fuzzy and incomplete information, it has defects in dealing with fuzzy, incomplete and inconsistent information. In order to deal with these issues, Smarandache [17] and Wang et al. [18] proposed the single-valued neutrosophic set—the truth–membership, indeterminacy–membership and falsity–membership degree are real numbers in the unit interval $[0, 1]$. In recent years, scholars have paid attention to studying the single-valued neutrosophic set. Smarandache [19] proposed n-norm and n-conorm in neutrosophic logic. Alkhazaleh [20] has provided some norms and conorms based on the neutrosophic set. Zhang et al. [21] provided new inclusion relations for neutrosophic sets. Hu and Zhang [22] constructed the residuated lattices based on the neutrosophic t-norms and neutrosophic residual implications. Zhao et al. [23] studied reverse triple I algorithms based on the single-valued neutrosophic fuzzy inference. So far, there is only a little research on the fuzzy reasoning method based on single-valued neutrosophic sets. Therefore, we studied the fuzzy reasoning triple I method based on a class single-valued neutrosophic triangular norm.

The structure of this paper is as follows: some basic concepts for single-valued neutrosophic sets are reviewed in Section 2. In Section 3, we give fuzzy inference triple I principles based on left-continuous single-valued neutrosophic t-representable t-norms, and the corresponding solutions of single-valued neutrosophic triple I methods. In Section 4, the robustness of the triple I method based on left-continuous single-valued neutrosophic t-representable t-norm is investigated. Finally, the conclusions are given in Section 5.

2. Preliminaries

In this section, we review some basic concepts for the single-valued neutrosophic set, single-valued neutrosophic t-norm and single-valued neutrosophic residual implication, which will be used in this article.

Definition 1 ([18]). Let X be a universal set. A single-valued neutrosophic set A on X is characterized by three functions, i.e., a truth–membership function $t_A(x)$, an indeterminacy–membership function $i_A(x)$, and a falsity–membership function $f_A(x)$. A single-valued neutrosophic set A can be defined as follows:

$$A = \{ \langle x, t_A(x), i_A(x), f_A(x) \rangle \mid x \in X \},$$

where $t_A(x), i_A(x), f_A(x) \in [0, 1]$ and satisfy the condition $0 \leq t_A(x) + i_A(x) + f_A(x) \leq 3$ for each x in X . The family of all single-valued neutrosophic sets on X is denoted by $SVNS(X)$.

The set of all single-valued neutrosophic numbers denoted by $SVNN$, i.e., $SVNN = \{ \langle t, i, f \rangle \mid t, i, f \in [0, 1] \}$. Let $\alpha = \langle t_\alpha, i_\alpha, f_\alpha \rangle, \beta = \langle t_\beta, i_\beta, f_\beta \rangle \in SVNN$, an ordering on $SVNN$ as $\alpha \leq \beta$ if and only if $t_\alpha \leq t_\beta, i_\alpha \geq i_\beta, f_\alpha \geq f_\beta, \alpha = \beta$ iff $\alpha \leq \beta$ and $\beta \leq \alpha$. Obviously, $\alpha \wedge \beta = \langle t_\alpha \wedge t_\beta, i_\alpha \vee i_\beta, f_\alpha \vee f_\beta \rangle, \alpha \vee \beta = \langle t_\alpha \vee t_\beta, i_\alpha \wedge i_\beta, f_\alpha \wedge f_\beta \rangle, \bigwedge_{i \in I} \alpha_i = \langle \bigwedge_{i \in I} t_{\alpha_i}, \bigvee_{i \in I} i_{\alpha_i}, \bigvee_{i \in I} f_{\alpha_i} \rangle, \bigvee_{i \in I} \alpha_i = \langle \bigvee_{i \in I} t_{\alpha_i}, \bigwedge_{i \in I} i_{\alpha_i}, \bigwedge_{i \in I} f_{\alpha_i} \rangle, 0^* = \langle 0, 1, 1 \rangle$ and $1^* = \langle 1, 0, 0 \rangle$ are the smallest element and the greatest element in $SVNN$, respectively. It is easy to verify that $(SVNN, \leq)$ is a complete lattice [18].

Definition 2 ([23]). The function $\mathcal{T}: SVNN \times SVNN \rightarrow SVNN$ defined by $\mathcal{T}(\alpha, \beta) = \langle T(t_\alpha, t_\beta), S(i_\alpha, i_\beta), S(f_\alpha, f_\beta) \rangle$ is a single-valued neutrosophic t-norm, which is called a single-valued neutrosophic t-representable t-norm, where T is a t-norm and S is its dual t-conorm on $[0, 1]$. $\mathcal{T}$ is called a left-continuous single-valued neutrosophic t-representable t-norm if T is left-continuous and S is right-continuous.

Definition 3 ([23]). A single-valued neutrosophic residual implication is defined by $\mathcal{R}_{\mathcal{T}}(\alpha, \beta) = \sup\{\gamma \in \text{SVNN} \mid \mathcal{T}(\gamma, \alpha) \leq \beta\}$, $\forall \alpha, \beta \in \text{SVNN}$, where $\mathcal{T}$ is a left-continuous single-valued neutrosophic t -representable t -norm.

Proposition 1 ([23]). Let $\alpha = \langle t_{\alpha}, i_{\alpha}, f_{\alpha} \rangle$, $\beta = \langle t_{\beta}, i_{\beta}, f_{\beta} \rangle \in \text{SVNN}$, then $\mathcal{R}_{\mathcal{T}_L}(\alpha, \beta) = \langle R_T(t_{\alpha}, t_{\beta}), R_S(i_{\beta}, i_{\alpha}), R_S(f_{\beta}, f_{\alpha}) \rangle$, which is the single-valued neutrosophic residual implication induced by left-continuous single-valued neutrosophic t -representable t -norm, where R_T is the residual implication induced by the left-continuous t -norm T ; R_S is the coresidual implication induced by the right-continuous t -conorm S .

Example 1 ([23]). The following are three important single-valued neutrosophic t -representable t -norms and their residual implications.

(1) The single-valued neutrosophic Łukasiewicz t -norm and its residual implication:

$$\mathcal{T}_L(\alpha, \beta) = \langle (t_{\alpha} + t_{\beta} - 1) \vee 0, (i_{\alpha} + i_{\beta}) \wedge 1, (f_{\alpha} + f_{\beta}) \wedge 1 \rangle.$$

$$\mathcal{R}_{\mathcal{T}_L}(\alpha, \beta) = \langle 1 \wedge (1 - t_{\alpha} + t_{\beta}), (i_{\beta} - i_{\alpha}) \vee 0, (f_{\beta} - f_{\alpha}) \vee 0 \rangle.$$

(2) The single-valued neutrosophic Gougen t -norm and its residual implication:

$$\mathcal{T}_{Go}(\alpha, \beta) = \langle t_{\alpha}t_{\beta}, i_{\alpha} + i_{\beta} - i_{\alpha}i_{\beta}, f_{\alpha} + f_{\beta} - f_{\alpha}f_{\beta} \rangle.$$

$$\mathcal{R}_{\mathcal{T}_{Go}}(\alpha, \beta) = \begin{cases} \langle 1, 0, 0 \rangle, & \text{if } t_{\alpha} \leq t_{\beta}, i_{\beta} \leq i_{\alpha}, f_{\beta} \leq f_{\alpha}, \\ \langle 1, 0, \frac{f_{\beta} - f_{\alpha}}{1 - f_{\alpha}} \rangle, & \text{if } t_{\alpha} \leq t_{\beta}, i_{\beta} \leq i_{\alpha}, f_{\alpha} < f_{\beta}, \\ \langle 1, \frac{i_{\beta} - i_{\alpha}}{1 - i_{\alpha}}, 0 \rangle, & \text{if } t_{\alpha} \leq t_{\beta}, i_{\alpha} < i_{\beta}, f_{\beta} \leq f_{\alpha}, \\ \langle 1, \frac{i_{\beta} - i_{\alpha}}{1 - i_{\alpha}}, \frac{f_{\beta} - f_{\alpha}}{1 - f_{\alpha}} \rangle, & \text{if } t_{\alpha} \leq t_{\beta}, i_{\alpha} < i_{\beta}, f_{\alpha} < f_{\beta}, \\ \langle \frac{t_{\beta}}{t_{\alpha}}, 0, 0 \rangle, & \text{if } t_{\beta} < t_{\alpha}, i_{\beta} \leq i_{\alpha}, f_{\beta} \leq f_{\alpha}, \\ \langle \frac{t_{\beta}}{t_{\alpha}}, 0, \frac{f_{\beta} - f_{\alpha}}{1 - f_{\alpha}} \rangle, & \text{if } t_{\beta} < t_{\alpha}, i_{\beta} \leq i_{\alpha}, f_{\alpha} < f_{\beta}, \\ \langle \frac{t_{\beta}}{t_{\alpha}}, \frac{i_{\beta} - i_{\alpha}}{1 - i_{\alpha}}, 0 \rangle, & \text{if } t_{\beta} < t_{\alpha}, i_{\alpha} < i_{\beta}, f_{\beta} \leq f_{\alpha}, \\ \langle \frac{t_{\beta}}{t_{\alpha}}, \frac{i_{\beta} - i_{\alpha}}{1 - i_{\alpha}}, \frac{f_{\beta} - f_{\alpha}}{1 - f_{\alpha}} \rangle, & \text{if } t_{\beta} < t_{\alpha}, i_{\alpha} < i_{\beta}, f_{\alpha} < f_{\beta}. \end{cases}$$

(3) The single-valued neutrosophic Gödel t -norm and its residual implication:

$$\mathcal{T}_G(\alpha, \beta) = \langle t_{\alpha} \wedge t_{\beta}, i_{\alpha} \vee i_{\beta}, f_{\alpha} \vee f_{\beta} \rangle.$$

$$\mathcal{R}_{\mathcal{T}_G}(\alpha, \beta) = \begin{cases} \langle 1, 0, 0 \rangle, & \text{if } t_{\alpha} \leq t_{\beta}, i_{\beta} \leq i_{\alpha}, f_{\beta} \leq f_{\alpha}, \\ \langle 1, 0, f_{\beta} \rangle, & \text{if } t_{\alpha} \leq t_{\beta}, i_{\beta} \leq i_{\alpha}, f_{\alpha} < f_{\beta}, \\ \langle 1, i_{\beta}, 0 \rangle, & \text{if } t_{\alpha} \leq t_{\beta}, i_{\alpha} < i_{\beta}, f_{\beta} \leq f_{\alpha}, \\ \langle 1, i_{\beta}, f_{\beta} \rangle, & \text{if } t_{\alpha} \leq t_{\beta}, i_{\alpha} < i_{\beta}, f_{\alpha} < f_{\beta}, \\ \langle t_{\beta}, 0, 0 \rangle, & \text{if } t_{\beta} < t_{\alpha}, i_{\beta} \leq i_{\alpha}, f_{\beta} \leq f_{\alpha}, \\ \langle t_{\beta}, 0, f_{\beta} \rangle, & \text{if } t_{\beta} < t_{\alpha}, i_{\beta} \leq i_{\alpha}, f_{\alpha} < f_{\beta}, \\ \langle t_{\beta}, i_{\beta}, 0 \rangle, & \text{if } t_{\beta} < t_{\alpha}, i_{\alpha} < i_{\beta}, f_{\beta} \leq f_{\alpha}, \\ \langle t_{\beta}, i_{\beta}, f_{\beta} \rangle, & \text{if } t_{\beta} < t_{\alpha}, i_{\alpha} < i_{\beta}, f_{\alpha} < f_{\beta}. \end{cases}$$

3. Single-Valued Neutrosophic Fuzzy Inference Triple I Method

Definition 4. (Single-valued neutrosophic fuzzy inference triple I principle for FMP). Suppose that $\mathcal{R}$ is a single-valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t -representable t -norm $\mathcal{T}$, $A, A^* \in \text{SVNS}(X)$ and $B \in \text{SVNS}(Y)$. Let $P(x, y) = \mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), 1^*))$, and $\mathbf{B}(A, B, A^*) = \{C \in \text{SVNS}(Y) \mid \mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(A^*(x), C(y))) = P(x, y), x \in X, y \in Y\}$. If there exist the smallest element of the set $\mathbf{B}(A, B, A^*)$ (denoted by B^*), then B^* is called the single-valued neutrosophic fuzzy inference triple I solution for FMP.

Definition 5. (Single-valued neutrosophic fuzzy inference triple I principle for FMT). Suppose that $\mathcal{R}$ is a single-valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t -representable t -norm $\mathcal{T}$. $A \in \text{SVNS}(X)$ and $B, B^* \in \text{SVNS}(Y)$. Let $Q(x, y) = \mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(0^*, B^*(x)))$, and $\mathbf{A}(A, B, B^*) = \{D \in \text{SVNS}(X) \mid \mathcal{R}(\mathcal{R}(A(x), B(y)), \mathcal{R}(D(x), B^*(x))) = Q(x, y), x \in X, y \in Y\}$. If there exist the greatest element of the set $\mathbf{A}(A, B, B^*)$ (denoted by A^*), then A^* is called the single-valued neutrosophic fuzzy inference triple I solution for FMT.

Theorem 1. Let $A, A^* \in \text{SVNS}(X)$, $B \in \text{SVNS}(Y)$, $\mathcal{R}$ be single-valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t -representable t -norm $\mathcal{T}$, then the single-valued neutrosophic fuzzy inference triple I solution B^* of FMP is $B^*(y) = \sup_{x \in X} \mathcal{T}(A^*(x), \mathcal{R}(A(x), B(y)))$ ($\forall y \in Y$).

Theorem 2. Let $A \in \text{SVNS}(X)$, $B, B^* \in \text{SVNS}(Y)$, $\mathcal{R}$ be single-valued neutrosophic residual implication induced by a left-continuous single-valued neutrosophic t -representable t -norm $\mathcal{T}$, then the single-valued neutrosophic fuzzy inference triple I solution A^* of FMT is $A^*(x) = \bigwedge_{y \in Y} \mathcal{R}(\mathcal{R}(A(x), B(y)), B^*(y))$ ($\forall x \in X$).

Definition 6 ([4]). A method for FMP is called recoverable if $A^* = A$ implies $B^* = B$. similarly, a method for FMT is called recoverable if $B^* = B$ implies $A^* = A$.

Theorem 3. The single-valued neutrosophic fuzzy inference triple I method for FMP is reductive if A is normal single-valued neutrosophic set.

Theorem 4. The single-valued neutrosophic fuzzy inference triple I method for FMT is reductive if single-valued neutrosophic residual implication $\mathcal{R}$ satisfy $\mathcal{R}(\mathcal{R}(A, 0^*), 0^*) = A$, and B is co-normal single-valued neutrosophic set.

4. Robustness of Single-Valued Neutrosophic Fuzzy Inference Triple I Method

Theorem 5. Let $A, B \in \text{SVNS}(X)$. $d : \text{SVNS}(X) \times \text{SVNS}(X) \rightarrow [0, 1]$ is defined $d(A, B) = \max \left\{ \bigvee_{x_i \in X} |t_A(x_i) - t_B(x_i)|, \bigvee_{x_i \in X} |i_A(x_i) - i_B(x_i)|, \bigvee_{x_i \in X} |f_A(x_i) - f_B(x_i)| \right\}$. Then, d is a metric on $\text{SVNS}(X)$, which is called a distance on $\text{SVNS}(X)$.

Definition 7. Suppose that $\mathfrak{F}$ is a n -tuple mapping: $\text{SVNN}^n \rightarrow \text{SVNN}$, $\forall \epsilon \in (0, 1)$. For any $A = \left\{ \langle t_{A_j}, i_{A_j}, f_{A_j} \rangle, j = 1, 2, \dots, n \right\} \in \text{SVNN}^n$, $\Delta_{\mathfrak{F}}(A, \epsilon) = \{d(\mathfrak{F}(A), \mathfrak{F}(A')) \mid A' \in \text{SVNN}^n, d(A, A') \leq \epsilon\}$ is called ϵ sensitivity of $\mathfrak{F}$ at point A . The biggest ϵ sensitivity of $\mathfrak{F}$ denoted by $\Delta_{\mathfrak{F}}(\epsilon) = \bigvee_{A \in \text{SVNN}^n} \Delta_{\mathfrak{F}}(A, \epsilon)$ is called ϵ sensitivity of $\mathfrak{F}$.

Definition 8. Let A and A' be two single-valued neutrosophic fuzzy sets on universal X . If $\|A - A'\| = \bigvee_{x \in X} d(A(x), A'(x)) \leq \epsilon$ for all $x \in X$, then A' is called ϵ -perturbation of A .

Theorem 6. Let A, A', B, B', A^* and A'^* be single-valued neutrosophic fuzzy sets. If $\|A - A'\| \leq \epsilon$, $\|B - B'\| \leq \epsilon$, $\|A^* - A'^*\| \leq \epsilon$, B^* and B'^* are the single-valued neutrosophic fuzzy inference triple I solutions of FMP(A, B, A^*) and FMP(A', B', A'^*) given in Theorem 3.1, respectively, then the ϵ sensitivity of the single-valued neutrosophic fuzzy inference triple I solution B^* for FMP is $\Delta_{B^*}(\epsilon) = \|B^* - B'^*\| \leq \Delta_{\mathcal{T}}(\Delta_{\mathcal{R}}(\epsilon))$.

Theorem 7. Let A, A', B, B', B^* and B'^* be single-valued neutrosophic fuzzy sets. If $\|A - A'\| \leq \epsilon$, $\|B - B'\| \leq \epsilon$, $\|B^* - B'^*\| \leq \epsilon$, A^* and A'^* are single-valued neutrosophic $\mathcal{R}$ -type triple I solutions of FMT (A, B, B^*) and FMT (A', B', B'^*) given in Theorem 3.2, respectively, then the ϵ sensitivity of the single-valued neutrosophic $\mathcal{R}$ -type triple I solution A^* for FMT is $\Delta_{A^*}(\epsilon) = \|A^* - A'^*\| \leq \Delta_{\mathcal{R}}(\Delta_{\mathcal{R}}(\epsilon))$.

5. Conclusions

In this paper, we extend the fuzzy inference triple I method on single-valued neutrosophic sets. The single-valued neutrosophic fuzzy inference triple I Principle for FMP and FMT are proposed. Moreover, the single-valued neutrosophic fuzzy inference triple I solutions for FMP and FMT are given, respectively. The reductivity and the robustness of the single-valued neutrosophic fuzzy inference triple I methods are studied.

The logical basis of a fuzzy inference method is very important. In future, we will consider building the strict logical foundation for the triple I method based on left-continuous single-valued neutrosophic t-representable t-norms, and bring the single-valued neutrosophic fuzzy inference method within the framework of logical semantics.

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