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Correction

Correction: Abramov, R. The Random Gas of Hard Spheres. *J* 2019, 2, 162–205

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In the published paper [1], we used the spatial correlation function $R(\sigma)$ of two spheres, each of diameter σ, to construct a closure to the BBGKY hierarchy of hard spheres. In the subsequent derivation of the fluid dynamics equations in [1], we relied upon the assumption that $R(\sigma) = 1 + O(\sigma)$. We later discovered that $R(\sigma)$ is the two-sphere cavity distribution function for hard spheres [2] in the constant sphere density limit, and thus is not $1 + O(\sigma)$. What follows below are the necessary corrections to the relevant equations in [1].

List of Changes

1. Equation (162) and the following sentence are changed as follows:

$$F^{(2)}(t,\boldsymbol{x},\boldsymbol{y},\boldsymbol{v},\boldsymbol{w}) \approx e^{-\lambda\Theta_{\alpha\sigma}(\sigma-\|\boldsymbol{x}-\boldsymbol{y}\|)} R^{(2)}((\boldsymbol{x}+\boldsymbol{y})/2, \|\boldsymbol{x}-\boldsymbol{y}\|) f(t,\boldsymbol{x},\boldsymbol{v}) f(t,\boldsymbol{y},\boldsymbol{w}), \tag{162}$$

   where $R^{(2)}$ depends parametrically on α and λ: $R^{(2)} = R^{(2)}_{\lambda,\alpha}$, and, in the case of a spatially nonuniform distribution, is also generally a function of the midpoint between $\boldsymbol{x}$ and $\boldsymbol{y}$.
2. The sentence following Equation (165) is changed as follows:
   "We can apply the formula above to our set-up by setting $z_1 = (\boldsymbol{x},\boldsymbol{v})$, $z_2 = (\boldsymbol{y},\boldsymbol{w})$, $F = F^{(2)}$, $\psi = e^{-\lambda\Theta_{\alpha\sigma}(\sigma-\|\boldsymbol{x}-\boldsymbol{y}\|)} R^{(2)}((\boldsymbol{x}+\boldsymbol{y})/2, \|\boldsymbol{x}-\boldsymbol{y}\|)$, and $p_1 = p_2 = \bar{F}^{(1)}$ from Equation (156)."
3. Equation (167) is changed as follows:

$$\begin{aligned}&\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{x}} = (K-1)\lambda\sigma^2 \int e^{-\lambda\Theta_\alpha(1-r)} \delta_\alpha(r-1)\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\Theta(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))\\ &\Big[R^{(2)}_{\lambda,\alpha}(\boldsymbol{x}+\sigma r\boldsymbol{n}/2,\sigma r) f(\boldsymbol{x},\boldsymbol{v}')f(\boldsymbol{x}+\sigma r\boldsymbol{n},\boldsymbol{w}') - R^{(2)}_{\lambda,\alpha}(\boldsymbol{x}-\sigma r\boldsymbol{n}/2,\sigma r) f(\boldsymbol{x},\boldsymbol{v})f(\boldsymbol{x}-\sigma r\boldsymbol{n},\boldsymbol{w})\Big] r^2 \mathrm{d}r\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}.\end{aligned} \tag{167}$$

4. The first sentence in Section 6.2 is changed as follows:
   "Above in Equation (167), we can formally assume that the contact zone is 'thin' that is, $\alpha \to 0$ so that, for the values of r for which $\delta_\alpha(r-1) > 0$, we have $f(\boldsymbol{x} \pm \sigma r\boldsymbol{n}) \to f(\boldsymbol{x} \pm \sigma\boldsymbol{n})$, $R^{(2)}_{\lambda,\alpha}(\boldsymbol{x} \pm \sigma r\boldsymbol{n}/2, \sigma r) \to R^{(2)}_{\lambda,0}(\boldsymbol{x} \pm \sigma\boldsymbol{n}/2, \sigma)$."
5. Equation (169) is changed as follows:

$$\begin{aligned}&\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{x}} = (K-1)\sigma^2\left(1-e^{-\lambda}\right)\int \boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\Theta(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))\\ &\Big[R^{(2)}_{\lambda,0}(\boldsymbol{x}+\sigma\boldsymbol{n}/2,\sigma) f(\boldsymbol{x},\boldsymbol{v}')f(\boldsymbol{x}+\sigma\boldsymbol{n},\boldsymbol{w}') - R^{(2)}_{\lambda,0}(\boldsymbol{x}-\sigma\boldsymbol{n}/2,\sigma) f(\boldsymbol{x},\boldsymbol{v})f(\boldsymbol{x}-\sigma\boldsymbol{n},\boldsymbol{w})\Big]\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}.\end{aligned} \tag{169}$$

6. Equation (170) is changed as follows, with the new sentence appended immediately after it:

$$\begin{aligned}\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f}{\partial \boldsymbol{x}} = (K-1)\sigma^2 &\int \boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\Theta(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))\\ &\Big[R(\boldsymbol{x}+\sigma\boldsymbol{n}/2) f(\boldsymbol{x},\boldsymbol{v}')f(\boldsymbol{x}+\sigma\boldsymbol{n},\boldsymbol{w}') - R(\boldsymbol{x}-\sigma\boldsymbol{n}/2) f(\boldsymbol{x},\boldsymbol{v})f(\boldsymbol{x}-\sigma\boldsymbol{n},\boldsymbol{w})\Big]\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w},\end{aligned} \tag{170}$$

where $R(\boldsymbol{x}) = R^{(2)}_{\infty,0}(\boldsymbol{x},\sigma)$ becomes the two-sphere cavity distribution function for hard spheres of diameter σ.

7. The last sentence before Section 7 is changed as follows:
"This sets $R(\boldsymbol{x} \pm \sigma\boldsymbol{n}/2) = 1$, $f(\boldsymbol{x} \pm \sigma\boldsymbol{n}) = f(\boldsymbol{x},\boldsymbol{v})\dots$"
8. Equation (172) is changed as follows:

$$\frac{\partial g}{\partial t} + \boldsymbol{v}\cdot\frac{\partial g}{\partial \boldsymbol{x}} = \frac{K-1}{K}\frac{6}{\pi\rho_{sp}\sigma}\int \boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big) \Big[R(\boldsymbol{x}+\sigma\boldsymbol{n}/2)g(\boldsymbol{x},\boldsymbol{v}')g(\boldsymbol{x}+\sigma\boldsymbol{n},\boldsymbol{w}') - R(\boldsymbol{x}-\sigma\boldsymbol{n}/2)g(\boldsymbol{x},\boldsymbol{v})g(\boldsymbol{x}-\sigma\boldsymbol{n},\boldsymbol{w})\Big]\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}. \tag{172}$$

9. Equation (173) is changed as follows:

$$\frac{\partial g}{\partial t} + \boldsymbol{v}\cdot\frac{\partial g}{\partial \boldsymbol{x}} = \frac{6}{\pi\rho_{sp}\sigma}\int \boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big) \Big[R(\boldsymbol{x}+\sigma\boldsymbol{n}/2)g(\boldsymbol{x},\boldsymbol{v}')g(\boldsymbol{x}+\sigma\boldsymbol{n},\boldsymbol{w}') - R(\boldsymbol{x}-\sigma\boldsymbol{n}/2)g(\boldsymbol{x},\boldsymbol{v})g(\boldsymbol{x}-\sigma\boldsymbol{n},\boldsymbol{w})\Big]\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}. \tag{173}$$

10. Equation (175), together with the preceding sentence, are changed as follows:
We also note that

$$R(\boldsymbol{x}\pm\sigma\boldsymbol{n}/2)g(\boldsymbol{x},\boldsymbol{v})g(\boldsymbol{x}\pm\sigma\boldsymbol{n},\boldsymbol{w}) = R(\boldsymbol{x})g(\boldsymbol{x},\boldsymbol{v})g(\boldsymbol{x},\boldsymbol{w}) \pm \sigma\sqrt{R(\boldsymbol{x})}g(\boldsymbol{x},\boldsymbol{v})\boldsymbol{n}\cdot\tfrac{\partial}{\partial \boldsymbol{x}}\Big(\sqrt{R(\boldsymbol{x})}g(\boldsymbol{x},\boldsymbol{w})\Big) + \dots \tag{175}$$

11. Equation (180) is changed as follows:

$$\begin{aligned}\tfrac{\partial g_0}{\partial t} + \boldsymbol{v}\cdot\tfrac{\partial g_0}{\partial \boldsymbol{x}} = \tfrac{6}{\pi\rho_{sp}}\int \boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\Big[R\big(g_0(\boldsymbol{v}')g_1(\boldsymbol{w}') + g_1(\boldsymbol{v}')g_0(\boldsymbol{w}') - \\ -g_0(\boldsymbol{v})g_1(\boldsymbol{w}) - g_1(\boldsymbol{v})g_0(\boldsymbol{w})\big) + \boldsymbol{n}\cdot\sqrt{R}\left(g_0(\boldsymbol{v}')\tfrac{\partial(\sqrt{R}g_0(\boldsymbol{w}'))}{\partial \boldsymbol{x}} + g_0(\boldsymbol{v})\tfrac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial \boldsymbol{x}}\right)\Big]\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}.\end{aligned} \tag{180}$$

12. Equations (185a) and (185b) are changed as follows:

$$\mathcal{C}[\boldsymbol{v}] = \frac{6}{\pi\rho_{sp}}\int(\boldsymbol{v}-\boldsymbol{v}')\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\boldsymbol{n}\cdot\frac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial \boldsymbol{x}}\sqrt{R}g_0(\boldsymbol{v})\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}, \tag{185a}$$

$$\mathcal{C}[\|\boldsymbol{v}\|^2] = \tfrac{6}{\pi\rho_{sp}}\int\big(\|\boldsymbol{v}\|^2 - \|\boldsymbol{v}'\|^2\big)\,\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\boldsymbol{n}\cdot\tfrac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial \boldsymbol{x}}\sqrt{R}g_0(\boldsymbol{v})\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}. \tag{185b}$$

13. Equations (187a) and (187b) are changed as follows:

$$\mathcal{C}[\boldsymbol{v}] = -\frac{6}{\pi\rho_{sp}}\int\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)^2\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\boldsymbol{n}\cdot\frac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial \boldsymbol{x}}\sqrt{R}g_0(\boldsymbol{v})\boldsymbol{n}\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}, \tag{187a}$$

$$\mathcal{C}[\|\boldsymbol{v}\|^2] = -\tfrac{6}{\pi\rho_{sp}}\int\big(\boldsymbol{n}\cdot(\boldsymbol{w}+\boldsymbol{v})\big)\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)^2\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\boldsymbol{n}\cdot\tfrac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial \boldsymbol{x}}\sqrt{R}g_0(\boldsymbol{v})\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}. \tag{187b}$$

14. Equation (188) is changed as follows:

$$\mathcal{C}[\boldsymbol{v}] = -\frac{4}{\rho_{sp}}\frac{\partial}{\partial \boldsymbol{x}}\left(R\rho^2\theta\right), \qquad \mathcal{C}[\|\boldsymbol{v}\|^2] = -\frac{8}{\rho_{sp}}\frac{\partial}{\partial \boldsymbol{x}}\cdot\left(R\rho^2\theta\boldsymbol{u}\right). \tag{188}$$

15. Equations (189a) and (189b) are changed as follows:

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}}\cdot(\rho\boldsymbol{u}) = 0, \qquad \frac{\partial(\rho\boldsymbol{u})}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}}\cdot\left(\rho\left(\boldsymbol{u}\boldsymbol{u}^T + \left(1 + \frac{4\rho}{\rho_{sp}}R\right)\theta\boldsymbol{I}\right)\right) = 0, \tag{189a}$$

$$\frac{\partial(\rho\epsilon)}{\partial t} + \frac{\partial}{\partial \boldsymbol{x}}\cdot\left(\rho\left(\epsilon + \left(1 + \frac{4\rho}{\rho_{sp}}R\right)\theta\right)\boldsymbol{u}\right) = 0. \tag{189b}$$

16. Equations (197a) and (197b) are changed as follows:

$$\frac{6}{\pi\rho_{sp}}\int \big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)^2\left(\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\boldsymbol{n}\boldsymbol{n}^T+\boldsymbol{n}(\boldsymbol{v}-\boldsymbol{u})^T+(\boldsymbol{v}-\boldsymbol{u})\boldsymbol{n}^T\right)\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)$$
$$\left(R\big(g_0(\boldsymbol{v})g_1(\boldsymbol{w})+g_1(\boldsymbol{v})g_0(\boldsymbol{w})\big)-\boldsymbol{n}\cdot\sqrt{R}g_0(\boldsymbol{v})\frac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}}\right)\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}=$$
$$=\rho\theta\left(\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}+\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}^T-\frac{2}{3}\left(\frac{\partial}{\partial\boldsymbol{x}}\cdot\boldsymbol{u}\right)\boldsymbol{I}\right)-\frac{8}{3\rho_{sp}}R\rho^2\theta\left(\frac{\partial}{\partial\boldsymbol{x}}\cdot\boldsymbol{u}\right)\boldsymbol{I}, \quad (197a)$$

$$\frac{6}{\pi\rho_{sp}}\int(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\Big[(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\boldsymbol{n}+(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))(\boldsymbol{I}+2\boldsymbol{n}\boldsymbol{n}^T)(\boldsymbol{v}-\boldsymbol{u})+$$
$$+\left(\|\boldsymbol{v}-\boldsymbol{u}\|^2\boldsymbol{I}+2(\boldsymbol{v}-\boldsymbol{u})(\boldsymbol{v}-\boldsymbol{u})^T\right)\boldsymbol{n}\Big]\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big) \quad (197b)$$
$$\left(R\big(g_0(\boldsymbol{v})g_1(\boldsymbol{w})+g_1(\boldsymbol{v})g_0(\boldsymbol{w})\big)-\boldsymbol{n}\cdot\sqrt{R}g_0(\boldsymbol{v})\frac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}}\right)\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}=5\rho\theta\frac{\partial\theta}{\partial\boldsymbol{x}}-\frac{20}{\rho_{sp}}\theta\frac{\partial(R\rho^2\theta)}{\partial\boldsymbol{x}}.$$

17. Equations (198a)–(198d) are changed as follows:

$$\int\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)^2\left(\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\boldsymbol{n}\boldsymbol{n}^T+\boldsymbol{n}(\boldsymbol{v}-\boldsymbol{u})^T+(\boldsymbol{v}-\boldsymbol{u})\boldsymbol{n}^T\right)$$
$$\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)R\left(g_0(\boldsymbol{v})g_1(\boldsymbol{w})+g_1(\boldsymbol{v})g_0(\boldsymbol{w})\right)\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}=-\frac{16\sqrt{\pi}}{5}R\rho^2\sqrt{\theta}\boldsymbol{S}, \quad (198a)$$

$$\int\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)^2\left(\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\boldsymbol{n}\boldsymbol{n}^T+\boldsymbol{n}(\boldsymbol{v}-\boldsymbol{u})^T+(\boldsymbol{v}-\boldsymbol{u})\boldsymbol{n}^T\right)$$
$$\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\boldsymbol{n}\cdot\sqrt{R}g_0(\boldsymbol{v})\frac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}}\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}=\frac{4\pi}{15}R\rho^2\theta\left(\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}+\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}^T+\left(\frac{\partial}{\partial\boldsymbol{x}}\cdot\boldsymbol{u}\right)\boldsymbol{I}\right), \quad (198b)$$

$$\int(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\Big[(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\boldsymbol{n}+(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))(\boldsymbol{I}+2\boldsymbol{n}\boldsymbol{n}^T)(\boldsymbol{v}-\boldsymbol{u})+$$
$$+\left(\|\boldsymbol{v}-\boldsymbol{u}\|^2\boldsymbol{I}+2(\boldsymbol{v}-\boldsymbol{u})(\boldsymbol{v}-\boldsymbol{u})^T\right)\boldsymbol{n}\Big]\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)$$
$$R\left(g_0(\boldsymbol{v})g_1(\boldsymbol{w})+g_1(\boldsymbol{v})g_0(\boldsymbol{w})\right)\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}=-\frac{64\sqrt{\pi}}{15}R\rho^2\sqrt{\theta}\boldsymbol{q}, \quad (198c)$$

$$\int(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\Big[(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\boldsymbol{n}+(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))(\boldsymbol{I}+2\boldsymbol{n}\boldsymbol{n}^T)(\boldsymbol{v}-\boldsymbol{u})+$$
$$+\left(\|\boldsymbol{v}-\boldsymbol{u}\|^2\boldsymbol{I}+2(\boldsymbol{v}-\boldsymbol{u})(\boldsymbol{v}-\boldsymbol{u})^T\right)\boldsymbol{n}\Big]\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)$$
$$\boldsymbol{n}\cdot\sqrt{R}g_0(\boldsymbol{v})\frac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}}\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}=\frac{10\pi}{3}\theta\frac{\partial(R\rho^2\theta)}{\partial\boldsymbol{x}}+2\pi R\rho^2\theta\frac{\partial\theta}{\partial\boldsymbol{x}}. \quad (198d)$$

18. Equations (199a) and (199b) are changed as follows:

$$\rho\boldsymbol{S}=\left(\frac{1}{R}+\frac{8\rho}{5\rho_{sp}}\right)[\rho\boldsymbol{S}]_B=-\left(\frac{1}{R}+\frac{8\rho}{5\rho_{sp}}\right)\mu\left(\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}+\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}^T-\frac{2}{3}\left(\frac{\partial}{\partial\boldsymbol{x}}\cdot\boldsymbol{u}\right)\boldsymbol{I}\right), \quad (199a)$$

$$\rho\boldsymbol{q}=\left(\frac{1}{R}+\frac{12\rho}{5\rho_{sp}}\right)[\rho\boldsymbol{q}]_B=-\left(\frac{1}{R}+\frac{12\rho}{5\rho_{sp}}\right)\frac{15}{4}\mu\frac{\partial\theta}{\partial\boldsymbol{x}}, \qquad \mu=\frac{5\sqrt{\pi}\rho_{sp}\sigma}{96}\sqrt{\theta}, \quad (199b)$$

19. Equations (201a) and (201b) are changed as follows:

$$\begin{aligned}&\tfrac{\partial(\rho\boldsymbol{u})}{\partial t}+\tfrac{\partial}{\partial\boldsymbol{x}}\cdot\left(\rho(\boldsymbol{u}\boldsymbol{u}^T+\theta\boldsymbol{I}+\sigma\boldsymbol{S})\right)=-\tfrac{4}{\rho_{sp}}\tfrac{\partial}{\partial\boldsymbol{x}}\cdot\left(R\rho^2\theta\right)+\\&+\tfrac{6\sigma}{\pi\rho_{sp}}\int(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\boldsymbol{n}\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\\&\sqrt{R}\left(-g_1(\boldsymbol{v})\boldsymbol{n}\cdot\tfrac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}}-g_0(\boldsymbol{v})\boldsymbol{n}\cdot\tfrac{\partial(\sqrt{R}g_1(\boldsymbol{w}))}{\partial\boldsymbol{x}}+\tfrac{1}{2}g_0(\boldsymbol{v})\boldsymbol{n}^T\tfrac{\partial^2(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}^2}\boldsymbol{n}\right)\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v},\end{aligned}\tag{201a}$$

$$\begin{aligned}&\tfrac{\partial(\rho\epsilon)}{\partial t}+\tfrac{\partial}{\partial\boldsymbol{x}}\cdot\left(\rho((\epsilon+\theta)\boldsymbol{u}+\sigma\boldsymbol{S}\boldsymbol{u}+\sigma\boldsymbol{q})\right)=-\tfrac{4}{\rho_{sp}}\tfrac{\partial}{\partial\boldsymbol{x}}\cdot\left(R\rho^2\theta\boldsymbol{u}\right)+\\&+\tfrac{3\sigma}{\pi\rho_{sp}}\int(\boldsymbol{n}\cdot(\boldsymbol{w}+\boldsymbol{v}))(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\\&\sqrt{R}\left(-g_1(\boldsymbol{v})\boldsymbol{n}\cdot\tfrac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}}-g_0(\boldsymbol{v})\boldsymbol{n}\cdot\tfrac{\partial(\sqrt{R}g_1(\boldsymbol{w}))}{\partial\boldsymbol{x}}+\tfrac{1}{2}g_0(\boldsymbol{v})\boldsymbol{n}^T\tfrac{\partial^2(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}^2}\boldsymbol{n}\right)\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}.\end{aligned}\tag{201b}$$

20. Equation (202), together with the preceding sentence, are changed as follows:
Above, we take advantage of the fact that, for $\psi(\boldsymbol{v})=\boldsymbol{v}$ or $\psi(\boldsymbol{v})=\|\boldsymbol{v}\|^2$,

$$\int(\psi(\boldsymbol{v}')-\psi(\boldsymbol{v}))\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)(\boldsymbol{n}\boldsymbol{n}^T)g_0(\boldsymbol{v})g_0(\boldsymbol{w})\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}=0.\tag{202}$$

21. Equations (203a)–(203d) are changed as follows:

$$\begin{aligned}&\int(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\boldsymbol{n}\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\\&\sqrt{R}\left(g_1(\boldsymbol{v})\boldsymbol{n}\cdot\frac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}}+g_0(\boldsymbol{v})\boldsymbol{n}\cdot\frac{\partial(\sqrt{R}g_1(\boldsymbol{w}))}{\partial\boldsymbol{x}}\right)\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}=\frac{4\pi}{15}\frac{\partial}{\partial\boldsymbol{x}}\cdot\left(R\rho^2\boldsymbol{S}\right),\end{aligned}\tag{203a}$$

$$\begin{aligned}&\int(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\boldsymbol{n}\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\boldsymbol{n}^T\frac{\partial^2(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}^2}\boldsymbol{n}\sqrt{R}g_0(\boldsymbol{v})\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}=\\&=\frac{8\sqrt{\pi}}{15}\frac{\partial}{\partial\boldsymbol{x}}\cdot\left(R\rho^2\sqrt{\theta}\left(\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}+\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}^T+\left(\frac{\partial}{\partial\boldsymbol{x}}\cdot\boldsymbol{u}\right)\boldsymbol{I}\right)\right),\end{aligned}\tag{203b}$$

$$\begin{aligned}&\int(\boldsymbol{n}\cdot(\boldsymbol{w}+\boldsymbol{v}))(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\\&\sqrt{R}\left(g_1(\boldsymbol{v})\boldsymbol{n}\cdot\tfrac{\partial(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}}+g_0(\boldsymbol{v})\boldsymbol{n}\cdot\tfrac{\partial(\sqrt{R}g_1(\boldsymbol{w}))}{\partial\boldsymbol{x}}\right)\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}=\tfrac{8\pi}{15}\tfrac{\partial}{\partial\boldsymbol{x}}\cdot\left(R\rho^2\left(\boldsymbol{S}\boldsymbol{u}+\tfrac{3}{2}\boldsymbol{q}\right)\right),\end{aligned}\tag{203c}$$

$$\begin{aligned}&\int(\boldsymbol{n}\cdot(\boldsymbol{w}+\boldsymbol{v}))(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v}))^2\Theta\big(\boldsymbol{n}\cdot(\boldsymbol{w}-\boldsymbol{v})\big)\boldsymbol{n}^T\frac{\partial^2(\sqrt{R}g_0(\boldsymbol{w}))}{\partial\boldsymbol{x}^2}\boldsymbol{n}\sqrt{R}g_0(\boldsymbol{v})\mathrm{d}\boldsymbol{n}\mathrm{d}\boldsymbol{w}\mathrm{d}\boldsymbol{v}=\\&=\frac{16\sqrt{\pi}}{15}\frac{\partial}{\partial\boldsymbol{x}}\cdot\left(R\rho^2\sqrt{\theta}\left[\left(\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}+\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}^T+\left(\frac{\partial}{\partial\boldsymbol{x}}\cdot\boldsymbol{u}\right)\boldsymbol{I}\right)\boldsymbol{u}+\frac{5}{2}\frac{\partial\theta}{\partial\boldsymbol{x}}\right]\right).\end{aligned}\tag{203d}$$

22. Equations (204a) and (204b) are changed as follows:

$$\begin{aligned}&\frac{\partial(\rho\boldsymbol{u})}{\partial t}+\frac{\partial}{\partial\boldsymbol{x}}\cdot\left(\rho(\boldsymbol{u}\boldsymbol{u}^T+\theta\boldsymbol{I})\right)=-\frac{4}{\rho_{sp}}\frac{\partial}{\partial\boldsymbol{x}}\cdot\left(R\rho^2\theta\right)-\\&-\sigma\frac{\partial}{\partial\boldsymbol{x}}\cdot\left(\left(1+\frac{8\rho}{5\rho_{sp}}R\right)\rho\boldsymbol{S}\right)+\frac{8\sigma}{5\sqrt{\pi}\rho_{sp}}\frac{\partial}{\partial\boldsymbol{x}}\cdot\left(R\rho^2\sqrt{\theta}\left(\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}+\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}}^T+\left(\frac{\partial}{\partial\boldsymbol{x}}\cdot\boldsymbol{u}\right)\boldsymbol{I}\right)\right),\end{aligned}\tag{204a}$$

$$\frac{\partial(\rho\epsilon)}{\partial t}+\frac{\partial}{\partial \boldsymbol{x}}\cdot(\rho(\epsilon+\theta)\boldsymbol{u})=-\frac{4}{\rho_{sp}}\frac{\partial}{\partial \boldsymbol{x}}\cdot\left(R\rho^2\theta\boldsymbol{u}\right)-$$
$$-\sigma\frac{\partial}{\partial \boldsymbol{x}}\cdot\left(\left(1+\frac{8\rho}{5\rho_{sp}}R\right)\rho\boldsymbol{S}\boldsymbol{u}\right)-\sigma\frac{\partial}{\partial \boldsymbol{x}}\cdot\left(\left(1+\frac{12\rho}{5\rho_{sp}}R\right)\rho\boldsymbol{q}\right)+$$
$$+\frac{8\sigma}{5\sqrt{\pi}\rho_{sp}}\frac{\partial}{\partial \boldsymbol{x}}\cdot\left(R\rho^2\sqrt{\theta}\left[\left(\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}+\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}^T+\left(\frac{\partial}{\partial \boldsymbol{x}}\cdot\boldsymbol{u}\right)\boldsymbol{I}\right)\boldsymbol{u}+\frac{5}{2}\frac{\partial\theta}{\partial \boldsymbol{x}}\right]\right). \tag{204b}$$

23. Equations (205a) and (205b) are changed as follows:

$$\frac{\partial(\rho\boldsymbol{u})}{\partial t}+\frac{\partial}{\partial \boldsymbol{x}}\cdot\left(\rho\left(\boldsymbol{u}\boldsymbol{u}^T+\left(1+\frac{4\rho}{\rho_{sp}}R\right)\theta\boldsymbol{I}\right)\right)=$$
$$=\frac{\partial}{\partial \boldsymbol{x}}\cdot\left(\mu\left(\left(\frac{1}{R}+a_1\right)\left(\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}+\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}^T\right)-\frac{2}{3}\left(\frac{1}{R}+a_2\right)\left(\frac{\partial}{\partial \boldsymbol{x}}\cdot\boldsymbol{u}\right)\boldsymbol{I}\right)\right), \tag{205a}$$

$$\frac{\partial(\rho\epsilon)}{\partial t}+\frac{\partial}{\partial \boldsymbol{x}}\cdot\left(\rho\left(\epsilon+\left(1+\frac{4\rho}{\rho_{sp}}R\right)\theta\right)\boldsymbol{u}\right)=\frac{15}{4}\frac{\partial}{\partial \boldsymbol{x}}\cdot\left(\mu\left(\frac{1}{R}+a_3\right)\frac{\partial\theta}{\partial \boldsymbol{x}}\right)+$$
$$+\frac{\partial}{\partial \boldsymbol{x}}\cdot\left(\mu\left(\left(\frac{1}{R}+a_1\right)\left(\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}+\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}^T\right)-\frac{2}{3}\left(\frac{1}{R}+a_2\right)\left(\frac{\partial}{\partial \boldsymbol{x}}\cdot\boldsymbol{u}\right)\boldsymbol{I}\right)\boldsymbol{u}\right). \tag{205b}$$

24. Equations (206a)–(206c) are changed as follows:

$$a_1\left(\frac{\rho}{\rho_{sp}}\right)=\frac{16\rho}{5\rho_{sp}}\left(1+\frac{4\rho}{5\rho_{sp}}R\left(1+\frac{12}{\pi}\right)\right), \tag{206a}$$

$$a_2\left(\frac{\rho}{\rho_{sp}}\right)=\frac{16\rho}{5\rho_{sp}}\left(1+\frac{4\rho}{5\rho_{sp}}R\left(1-\frac{18}{\pi}\right)\right), \tag{206b}$$

$$a_3\left(\frac{\rho}{\rho_{sp}}\right)=\frac{24\rho}{5\rho_{sp}}\left(1+\frac{2\rho}{15\rho_{sp}}R\left(9+\frac{32}{\pi}\right)\right). \tag{206c}$$

25. The paragraph preceding Equations (207a)–(207c), as well as Equations (207a)–(207c), are removed. In the next paragraph, the reference [2] (which is [46] in the updated manuscript) is provided for cavity distribution functions for hard spheres.

    The main results of [1] are unchanged, and the summary remains the same.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Abramov, R. The Random Gas of Hard Spheres. *J* **2019**, *2*, 162–205. [CrossRef]
2. Boublík, T. Hard-Sphere Radial Distribution Function from the Residual Chemical Potential. *Molec. Phys.* **2006**, *104*, 3425–3433. [CrossRef]