

Proceeding Paper

Evaluation of the Equivalent Moving Force Model for Lightweight Aluminum Footbridges in Simulating the Bridge Response under a Single-Pedestrian Walking Load [†]

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Abstract: Due to its high strength-to-weight ratio, corrosion resistance, extrudability, and recyclability, there is a growing demand for the use of aluminum for sustainable bridge constructions, especially for footbridges. Owing to their light weight and lively characteristics, vibration serviceability often governs the design of aluminum footbridges. To better design these bridges, it is necessary to accurately predict pedestrian-induced walking loads. To this end, the existing design codes around the world have adopted the periodic moving force model (MF) due to its simplicity. However, the appropriateness of the MF model is being questioned, mainly in capturing the effect of the human–structure interaction (HSI), which can be pertinent to the design behavior of aluminum footbridges. A biomechanical-based spring-mass-damper (SMD) model was developed in the literature to simulate the HSI phenomena, which has never been validated for aluminum footbridges. Moreover, to simplify the complexity associated with SMD models, the experimental moving force model (EMF) was developed that can capture the effect of the HSI in an equivalent sense. This study aims to evaluate the capability of the SMD and EMF models to capture the real behavior of aluminum footbridges. To do so, the vibration responses of two aluminum footbridges are simulated numerically as a single-degree-of-freedom (SDOF) system under single-pedestrian walking loads employing the SMD, EMF, and MF models, which are then compared and validated based on already-available experimental observations. Finally, recommendations are made for the most suitable model for the vibration prediction of aluminum footbridges.



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1. Introduction

Increasing demand for sustainable bridge constructions has led to a growing interest in the use of aluminum, owing to its remarkable properties such as high strength-to-weight ratio, corrosion resistance, extrudability, and recyclability. Uses of aluminum in the construction of footbridges, in particular, have gained significant attention due to their durability as well as lightweight nature. While the lightweight characteristic of such a bridge helps in its faster construction, it causes vibration problems, leading to serviceability issues. Vibration serviceability plays a critical role in the design of aluminum footbridges, necessitating accurate prediction of pedestrian-induced walking loads [1,2].

The accurate prediction of vibration response in aluminum footbridges presents complexities due to their inherent medium- to high-frequency characteristics and lightweight nature [1–4]. These characteristics result in the fundamental frequency of the bridge being outside the range of the first harmonic of the walking load, which has been extensively studied in the existing literature. Consequently, there is a higher likelihood of resonance occurring with the higher harmonics of the walking load rather than the first harmonic of the pacing frequency [1]. To address this need, Dey et al. [2] have recommended a

few modifications in the existing design codes that have commonly adopted the periodic moving force model (MF) due to its simplicity [5–7]. Although simplified and easy to be applied for design applications, there are still concerns regarding the appropriateness of the MF model, particularly in capturing the crucial aspect of the human–structure interaction (HSI), which significantly influences the design behavior of aluminum footbridges [2].

In the literature, a biomechanical-based spring-mass-damper (SMD) model has been developed to simulate the HSI phenomena [8], which are often very complex to be applied in design applications. Hence, an experimental moving force model (EMF) was developed to provide an equivalent yet simplified representation of the HSI effect on the bridge response [9]. The EMF model accounts for the HSI by estimating an equivalent damping ratio considering the pedestrian's dynamic properties. Nevertheless, the suitability and accuracy of both the SMD and EMF models in representing the behavior of aluminum footbridges with the potential to resonate with the higher harmonics of walking frequencies remain unexplored. This study aims to evaluate the capability of the SMD and EMF models to accurately capture the dynamic behavior of aluminum footbridges under pedestrian loads, while its scope is limited to single pedestrian-induced vertical walking loads only.

To reach the objective of this study, two aluminum footbridges are equated with an SDOF systems considering their associated first modal characteristic. Then, numerical simulations are conducted for analyzing the vibration responses of these footbridges under single pedestrian walking loads. The SMD, EMF, and MF models are employed in these simulations and their corresponding differential equations of motions are extracted; later, these equations are solved using the Newmark beta numerical technique with an appropriate time step in the MATLAB software, and the results are compared and validated against available experimental observations from those two footbridges as documented in [1]. Finally, this study provides recommendations for the most suitable model for predicting the vibration of aluminum footbridges.

2. Methodology

2.1. Past Experimental Study

The experimental investigation encompassed pedestrian walking tests conducted on two aluminum pony truss pedestrian bridges of different dynamic properties. The properties of the bridges are discussed in the next section. A series of walking tests including single and multiple pedestrians walking were conducted with different walking frequencies. Since the current study is limited to single-pedestrian walking, the experimental data, including pedestrian properties and measured acceleration on the two bridges, have been extracted. A brief overview of the bridge and pedestrian properties is presented below, while more detailed information on the experimental study can be found in [10].

2.1.1. Bridge Properties

The study involved tests where pedestrians walked on two different aluminum pony truss pedestrian bridges. The first bridge, measuring 12.2 m in length, comprised extruded aluminum members fastened to bolted joints. It had a mass of 982 kg, a damping ratio of 1.0%, and a fundamental frequency of 13.0 Hz. The second bridge, with a span of 22.9 m, was also modular and made of similar aluminum members. It weighed 1735 kg, possessed a damping ratio of 0.8%, and exhibited a natural frequency of 4.4 Hz.

2.1.2. Pedestrian Properties

In order to model pedestrian-induced walking loads and validate them with the experimental data available in [1], a test subject representing a pedestrian with mass of 70 kg has been selected. During the walking tests, the test subject walked in five different frequencies ranging from 1.670 Hz to 2.338 Hz, with an equal interval of 0.167 Hz. This frequency range represents non-resonant, near-resonant, and resonant scenarios for both footbridges, ensuring comprehensive coverage of different vibration-response conditions. The current study classifies these frequencies as non-resonant, near-resonant, and resonant

situations based on the percentage differences between the frequency of a harmonic of the walking load and the fundamental frequency of the footbridge, as it can be seen in Table 1. According to this classification, non-resonant situations corresponded to a difference of 100 to 10%, near-resonant situations represented a difference of 10% to 4%, and resonant situations indicated a difference of 4% to 0% between the frequency of excitation and fundamental frequency of the footbridge.

Table 1. Classification of non-resonant, near-resonant, and resonant scenarios.

Footbridge	$F_p^1 = 1.670 \text{ Hz}$	$F_p = 1.837 \text{ Hz}$	$F_p = 2.004 \text{ Hz}$	$F_p = 2.171 \text{ Hz}$	$F_p = 2.338 \text{ Hz}$
L = 22.9 m	Non-resonant	Non-resonant	Non-resonant	R-(H2) ²	NR-(H2)
L = 12.2 m	Non-resonant	Non-resonant	NR-(H6) ³	R-(H6)	NR-(H6)

F_p^1 : Pacing frequency of the pedestrian; R-(H2)²: resonant situation with harmonic 2. R-(H6)³: resonant situation with harmonic 6.

2.2. Numerical Simulation of Bridge Response

This section provides a brief description of the three types of walking-load models and the numerical simulation of bridge responses in vertical direction under each of these models, corresponding to the test subject walking at five different pacing frequencies, as described previously.

2.2.1. Moving Force (MF) Model

Blanchard et al. [11] conducted a study demonstrating that the vertical force exerted by walking pedestrians can be effectively represented using a periodic load model. This force corresponds to the force measured on a rigid surface when subjected to human walking excitation and can be decomposed into different harmonics. Mathematically, the force can be represented as:

$$P(t) = G + \sum_{i=1}^n G\alpha_i \sin(i2\pi f_s t + \Phi_i) \tag{1}$$

where $P(t)$ is the pedestrian’s weight in [N], α_i is the dynamic load factor (DLF) corresponding to the i th harmonic of the walking frequency, f_s is the pacing frequency of the pedestrian in [Hz], Φ_i is the phase shift of the i th harmonic, and n is the order number of the harmonic and total number of contributing harmonics, respectively. Extensive research has been dedicated to quantify the DLF values over time [11–13]; however, mostly for low-frequency bridges. In this study, the recommendations proposed by Dey et al. [3] have been adopted to determine the DLF values specifically applicable to lightweight bridges, which are as follows: $\alpha_1 = 0.37f_s - 0.42$; $\alpha_2 = 0.053$; $\alpha_3 = 0.042$, $\alpha_4 = 0.041$, and $\alpha_5 = 0.028$; and $\alpha_6 = 0.017$. Figure 1a presents the numerical model of the bridge and the MF that is simulated numerically to obtain the acceleration response of the bridge under MF model.

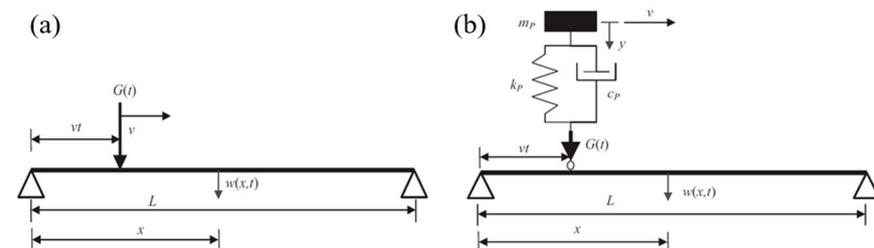


Figure 1. (a) The MF model and (b) the SMD model representing a pedestrian walking on a bridge modelled as a simply supported beam.

2.2.2. Spring-Mass-Damper (SMD) Model

The SMD model, depicted in Figure 1b, represents a linear single-degree-of-freedom (SDOF) mechanical system with a lumped mass, spring, and damper can be mathematically characterized to estimate the response of the bridge when in contact. However, there is a

limited body of research addressing the parametrization of the SMD model for bridges. Furthermore, inconsistencies exist in the values of SMD parameters across various studies, making it challenging to determine the appropriate parameters for the model [12]. In this study, the approach proposed by Ahmadi et al. [13] for calculating the modal damping ratio of the SMD model in resonant situations has been adopted. This approach allows for a clear distinction between the SMD parameters associated with resonant scenarios with one of the harmonics of walking load. It is worth noting that according to this work, properties of the footbridge are not taken to the account for calculating the SMD parameters. For non-resonant situations, the recommendation provided by Zhou et al. [14] has been utilized, which suggests a damping value of 0.3 and a natural frequency of 1.8 Hz for the SMD model. When dealing with near-resonant situations, the average of the parameter values from both non-resonant and resonant cases has been employed. The responses of the bridge under the SMD model with these parameters under different situations are simulated upon modelling the SMD model on a beam structure representing the bridge, as shown in Figure 1b. The summary of the dynamic parameters of the SMD model in different scenarios are reported in Table 2.

Table 2. Dynamic properties of the SMD model for resonant, near-resonant, and non-resonant situation for the specific test subject introduced by [13,14].

Situation	Mass (Kg)	Natural Frequency (Hz)	Stiffness (N/m)	Damping (N.S/m)
Resonant	70	2.171	13,011.7	371.1
Non-resonant	70	1.8	8944.6	307.6
Near-resonant	70	1.98	10,987.1	339.1

This is solved using the Newmark-Beta scheme to simulate the vibration response of the bridge for a given set of SMD parameters. Before the main analysis, the time step of the analysis was reduced to the extent that any further reduction in the time step will not change the vibration response of the bridge.

2.2.3. Experimental Moving Force Model

The Experimental Moving Force (EMF) is equivalent to the SMD model, where the vibration response of the reference SMD system as shown in Figure 2a is matched to that of the EMF system by adjusting the equivalent damping ratio of the bridge. This adjustment is determined based on the SMD-to-bridge frequency ratio and the pedestrian-to-bridge modal mass ratio [13]. By using the equivalent damping ratio for the bridge in the EMF model, the vibration responses of the bridge can be calculated, yielding results equivalent to those obtained from more demanding simulations using the SMD model. Such simplicity of the EMF model is typically preferred from a practitioner's perspective [9] and, hence, this model has been evaluated in this study and compared with the traditional MF model, as well as more complex SMD models.

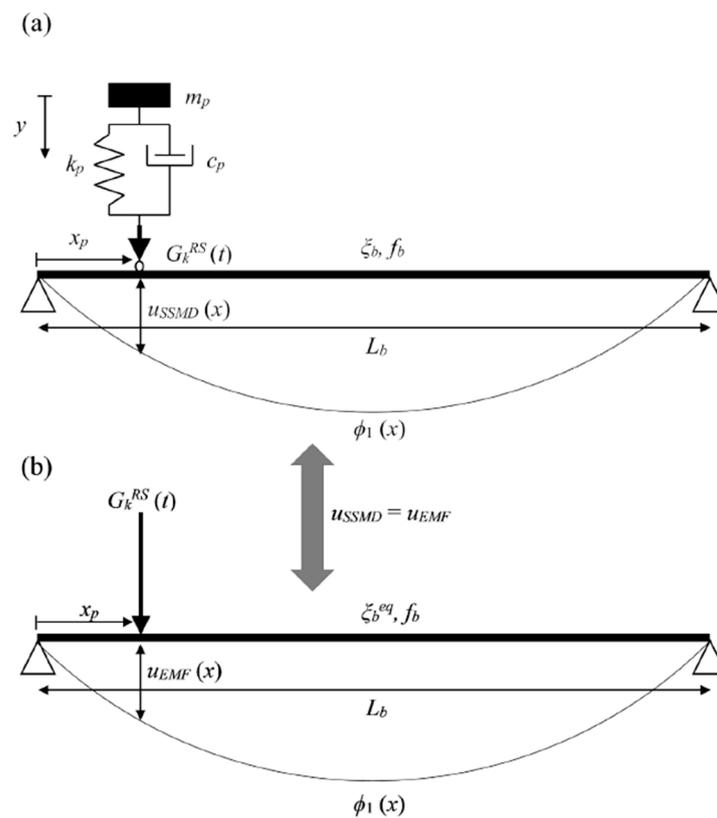


Figure 2. (a) Spring-mass-damper model and (b) and the associated equivalent experimental moving force model.

3. Results and Discussion

The maximum acceleration responses of the 22.9 m and 12.2 m bridge specimens under 70 kg test subject were simulated numerically for the above-mentioned three load models under different pacing frequencies representing slow (1.667 Hz) to fast (2.338 Hz) walking. As was previously mentioned, the equivalent SDOF system was generated considering first modal participation since it is proven that when one mode dominates, which often happens in footbridges, the response can be estimated sufficiently accurately using an SDOF modal equation for the appropriate mode. This is very often implemented in practice when checking footbridge vibration serviceability [15]. The numerical responses are compared with the measured acceleration of the bridges as extracted from [1] in Figures 3 and 4. The following sections discuss the observations of this comparison study for each of the bridge specimens.

3.1. 22.9 m Bridge Specimen

Figure 3 displays the maximum acceleration responses of the 22.9 m aluminum bridge specimen at its midpoint for different pacing frequencies of a single pedestrian weighing 70 kg. It compares the predicted responses by the MF, EMF, and SMD models with the measured ones. It is evident from the figure that all the walking models underestimate the bridge's vibration in non-resonant and near-resonant scenarios. However, in the resonant situation, the conventional MF model overestimates the bridge response (76% difference), as observed in the literature [1], while both the EMF and SMD models underestimate the responses in the resonant state with a maximum difference of 57% and 56%, respectively. Consequently, it can be concluded that none of the models accurately predict the vibration of the footbridge. Nonetheless, in non-resonant and near-resonant situations, the SMD model demonstrates better performance as compared with the MF or EMF models with a maximum of 54% difference in the prediction, as compared with the measured value. It is

believed that this improved accuracy is attributed to the SMD model’s ability to account for the interaction between the pedestrian and the bridge, which is a crucial factor in determining the dynamic behavior of aluminum footbridges.

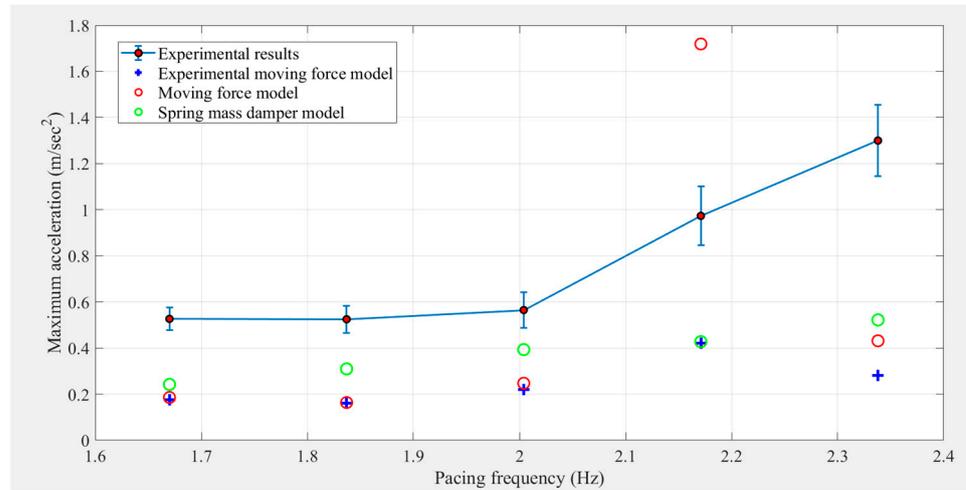


Figure 3. Simulated and measured maximum acceleration responses of the 22.9 m bridge specimen under single-pedestrian walking at different pacing frequencies.

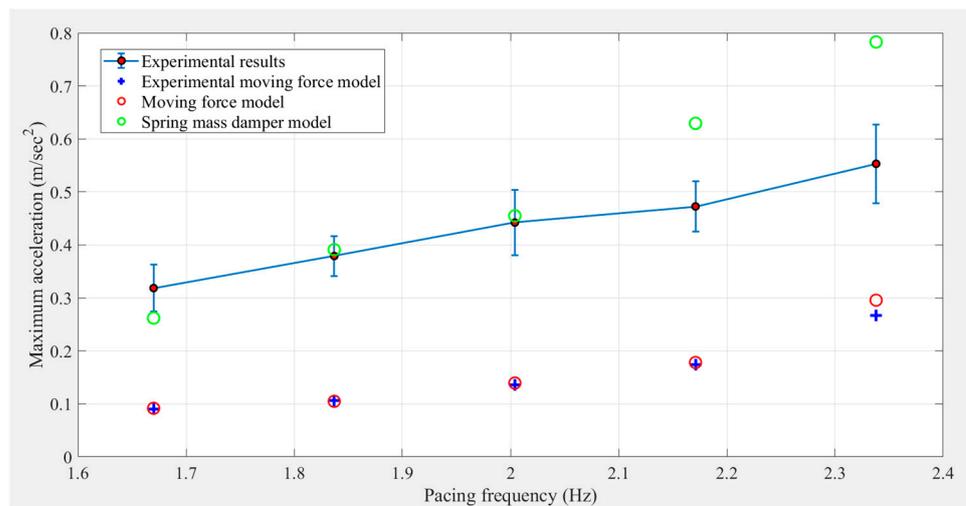


Figure 4. Simulated and measured maximum acceleration responses of the 12.2 m bridge specimen under single-pedestrian walking at different pacing frequencies.

Despite the initial expectation that modifying the damping of the coupled system (including the pedestrian and the footbridge) in the case of the EMF modelling approach would yield more compatible outcomes between measured and predicted values, there are negligible differences (a maximum of 11%) between the results of the EMF and MF models, especially in the case of non-resonant scenarios, while this difference increases in resonant cases since it is believed that damping has a significant impact on structures subjected to harmonic loading, particularly at resonances where the phase angle between the input force and displacement is 90 degrees, which is the same as the phase angle of the damping force. More interestingly, the EMF model predictions are closer to those of the SMD model as the bridge approaches resonance, while they are closer to those of the MF model, as it is away from the resonance. Nevertheless, further investigations are needed to investigate the reasoning for such behavior of the models and, accordingly, to calibrate the SMD and EMF models well for their improved performance in simulating the pedestrian-induced vibration response of aluminum footbridges.

3.2. 12.2 m Bridge Specimen

The results presented in Figure 4, attributed to the analysis conducted on a 12.2 m bridge specimen subjected to a single-pedestrian walking load, reveal an interesting finding. Both the MF and EMF models consistently underestimate the bridge vibration, regardless of whether it is in a resonant, non-resonant, or near-resonant state, in comparison to the experimental results. As observed earlier, the simulated responses using the MF and EMF models exhibit minimal differences for all scenarios (maximum of 9%). However, the notable difference in their trends between the two bridges is at the resonant case when there is significant difference between these two models for the 22.9 m bridge (75%), while there is a negligible difference between these two models for the 12.2 m bridge specimen (9%). This highlights the pressing need to investigate the EMF model further to exactly determine the reasons behinds such different observations in the case of these two bridges.

In contrast to the 22.9 m bridge specimen, the predictions made by the SMD model for the 12.2 m bridge exhibit closer proximity to the measured responses. This is particularly noticeable in non-resonant scenarios and the nearly resonant situation, specifically within the frequency range of 1.667 Hz to 2.004 Hz. The maximum difference between the predictions and measurements is 17% at 1.667 Hz. However, for the resonant frequency of 2.167 Hz and the nearby resonant frequency of 2.333 Hz, the SMD model tends to overestimate the acceleration response in the middle of the bridge, with a maximum difference of 23%. It is important to note that the trend observed in the SMD model predictions varies from underestimation at 1.667 Hz (non-resonant case with slow walking) to overestimation at 2.333 Hz (near-resonant case with fast walking). Further investigation is required to determine whether this trend is influenced by resonance, non-resonance, or simply the increase in pacing frequency. The inconsistent behavior of the SMD model for the two bridges suggests a lack of appropriate dynamic configurations for this specific type of footbridge, indicating the need for further study in the future.

4. Conclusions

This study has focused on evaluating the performance of MF, SMD, and EMF models in predicting the vibration response of two aluminum footbridges under single-pedestrian walking loads. Due to the unique properties of aluminum, such as light weight and lower damping, these footbridges experience excessive vibration, causing resonant conditions with the higher harmonics of the walking load. The accuracy of both the conventional MF model and the EMF model diminishes as the likelihood of resonance situations with higher harmonics of the walking load increases. This holds true regardless of whether the bridge is in a resonant, near-resonant, or non-resonant state. Nevertheless, further calibration of the EMF model is required for improved response predictions by this model, as the current modelling strategy has negligible improvement over the MF model.

On the other hand, the SMD model, which considers the human–structure interaction (HSI) phenomenon comprehensively as compared with the EMF model, yields relatively better results, mainly for the shorter bridge with potential resonance with higher harmonics than for the longer bridge. Nevertheless, the lack of calibration for the appropriate dynamic properties specific to lightweight aluminum footbridges hinders achieving an acceptable accuracy between experimental observations and numerical predictions by the SMD model. Moreover, further investigations are required to identify the reasons behind the different trends of behaviors by the SMD model for different bridges.

The future scope of this study includes an in-depth analysis of the SMD and EMF models based on more experimental observations following the calibration of these models for single-pedestrian walking loads and eventually extending these modelling strategies to crowd loads.

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